Bond Convenience Yields and Exchange Rate Dynamics

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Abstract

I propose a new explanation for the failure of Uncovered Interest Parity (UIP) that can rationalize not only the classic UIP puzzle, but also the evidence that the puzzle reverses direction at longer horizons. In the model, excess currency returns arise as compensation for endogenous fluctuations in bond convenience yields (i.e. liquidity value). Moreover, due to the interaction of monetary and fiscal policy, the dynamics of the equilibrium convenience yield are non-linear, which generates the reversal of the puzzle. The close relationship between UIP violations and monetary policy and government debt is also borne out by the data.

JEL Codes: F31, F41, F42, E43, E52, E63

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1 Introduction

Standard international models imply that the returns on default-free deposits across currencies are equalized. This is known as the Uncovered Interest Parity (UIP) condition and plays a central role in exchange rate determination in models. Yet, a long-standing puzzle in the literature is that this key condition fails in the data, and in fact there is significant forecastable time-variation in the differences in those returns. The typical finding underlying the classic UIP Puzzle is that an increase in domestic interest rates is associated with an increase in the excess return on the domestic currency over the foreign one.[1] However, recent evidence has shown that the puzzle is in fact more complex than commonly thought, as the comovement between interest rates and excess currency returns reverses direction at longer horizons, with high interest rates forecasting low excess returns at 4 to 7 year horizons.

This paper proposes a new mechanism that can rationalize not only the classic UIP puzzle, but also its reversal at longer horizons. It is based on endogenous fluctuations in bond convenience yields – i.e. the non-pecuniary benefit of holding safe and liquid assets that can serve as substitute for money, which is an important component of equilibrium bond yields in the data (Krishnamurthy and Vissing-Jorgensen (2012)). In the model, excess currency returns arise as a compensation for differences in the non-pecuniary values of bonds denominated in different currencies, and thus are equal the convenience yield differentials across countries. At times when the home convenience yield is low, investors require a compensating increase in the excess return of the home bond over the foreign one to compensate for its lower liquidity value. At the same time, a low home convenience yield prompts domestic investors to require a compensating increase in the bond's return over money, and thus an increase in the home interest rate. This generates a positive relationship between domestic interest rates and excess currency returns, and delivers the classic UIP puzzle. Moreover, the reversal of the direction of the puzzle is a result of the interaction between monetary and fiscal policy, which generates non-linear dynamics in the equilibrium convenience yield.

In particular, I extend an otherwise standard nominal two-country model by introducing a preference for liquidity over both money and bond holdings. Bonds serve as an imperfect substitute for money, and offer households both financial returns and liquidity services. The equilibrium convenience yield is the amount of interest investors are willing to forego in exchange for a bond's liquidity benefit, and is determined by the agents' consumption (i.e. volume of purchases), and by their holdings of liquid assets – money, and home and foreign government debt. As these equilibrium variables vary over the business cycle, so does

[1] See Fama (1984), Canova (1991), Canova and Marrinan (1993), Bekaert and Hodrick (1992), Backus et al. (1993), Hai et al. (1997), and the excellent surveys by Lewis (1995) and Engel (1996, 2013)

the convenience yield. Lastly, each country has a government that finances a fixed level of real expenditures by issuing nominal debt and levying lump-sum taxes. Monetary policy is set via a Taylor rule and tax policy via a Leeper (1991) rule, and the only exogenous shocks are standard productivity and monetary shocks.

In equilibrium, the excess currency return equals the convenience yield differential between the two countries, which is closely tied to the relative supply of home and foreign debt. Essentially, as one country's debt becomes scarcer relative to the other, its convenience yield increases *relative* to the other's convenience yield, and vice versa.[2] To illustrate, consider a contractionary home monetary shock that increases interest rates, and lowers inflation and output. The increase in the real interest rate and fall in output, and thus taxes, combine to increase home government debt, which lowers its convenience yield. In turn, the fall in the home convenience yield leads to a compensating increase in the equilibrium excess return of the domestic currency.[3] This generates the classic UIP puzzle of high interest rates being associated with high domestic currency returns.

In addition, the model can also explain the Engel (2016) empirical finding that excess currency returns, and thus UIP violations, change direction at longer horizons, an observation that he shows is at odds with the majority of existing UIP puzzle models. He finds that following an increase in the interest rate differential, currencies indeed fail to depreciate sufficiently in the short-run and thus earn high excess returns, but that they also display *excess* depreciation, and thus earn *low* excess returns at longer horizons. In other words, exchange rates exhibit a particular type of "delayed overshooting" where the eventual rate of depreciation exceeds the UIP benchmark. This phenomenon is a violation of UIP as well, but is the reverse of the classic puzzle, and had been previously unnoticed and unaddressed by the literature.

In my model, the switch in the direction of UIP violations is a result of non-linear dynamics in the equilibrium convenience yield, which are driven by the interaction between monetary and fiscal policy. In particular, when monetary policy is independent of fiscal considerations and tax policy is sluggish, there are feedback effects between the two that lead to cyclical (complex root) dynamics in debt that are also imparted to the equilibrium convenience yield. Going back to the example of a contractionary monetary shock, the rise in government debt prompts a persistent increase in taxes, which remain relatively high even as debt falls back towards steady state. This leads home debt to overshoot and fall below steady state before converging, but as it falls below steady state it now becomes relatively

[2]The link between debt supply and the convenience yield is also emphasized by the previous literature on bond convenience yields (Bansal and Coleman (1996), Krishnamurthy and Vissing-Jorgensen (2012)).

[3]I.e. investors are compensated for absorbing cross-country differences in the supply of liquid assets.

scarcer than foreign debt, and thus the convenience yield differential turns positive. As a result, the compensating excess return switches to the foreign currency, and this generates a change in the direction of UIP violations at longer horizons.

I analyze the mechanism in two steps. First, I derive analytical results in a stylized version of the model that distills it to its two key ingredients: endogenous convenience yield fluctuations and the interaction of monetary and fiscal policy. There, I analytically characterize the equilibrium dynamics of excess currency returns, and show that their changing nature arises due to feedback effects between a central bank focused on fighting inflation, and a persistent tax policy. In the second step, I return to the full model to examine the quantitative performance of the mechanism. I calibrate the model to standard parameters in the literature, and show that it closely matches the estimated empirical UIP violations at both short and long horizons, and that it delivers the appropriate, non-monotonic exchange rate dynamics.

The model also has a number of other appealing features. It implies that more hawkish monetary policy is associated with bigger and more cyclical UIP violations. This is corroborated by the data – I extend the original Bansal and Dahlquist (2000) analysis to medium-to-long horizons, and show that monetary policy independence is strongly associated with larger and more cyclical UIP violations. Moreover, thanks to the non-linear dynamics of convenience yields, the model can explain the Chinn (2006) findings that UIP holds better for long-term bonds, even if we assume that long-term bonds have the exact same non-pecuniary benefits as short-term bonds. Essentially, the equilibrium return on long-term investments across countries is equal to the sum of expected future short-term convenience yield differentials. But since the dynamics of the convenience yield is cyclical and changes signs, the sum of future expected convenience yield differentials is roughly zero, leading to no significant UIP violations in long-term bonds.

In addition, I provide direct empirical support for the mechanism by verifying its key implications in the data. First, I show that excess currency returns are closely related to fluctuations in government debt in the way implied by the model. Augmenting the standard UIP regression with the stock of debt, I find that increases in debt are indeed associated with statistically and economically significant increases in domestic currency returns. Importantly, I also find that the direction of the relationship reverses sign at longer horizons, conforming with the mechanism's explanation for the reversal in UIP violations. Second, I show that increases in credit spreads, a model-free measure of the convenience yield, are also associated with an increase in foreign currency returns, as implied by the basic mechanism.

The paper is related to both the empirical and the theoretical literature on the UIP puzzle, and to the literature on bond convenience yields. My empirical analysis confirms the original findings of Engel (2016) on the changing nature of UIP deviations, and follows up on them in two ways. First, I use a different empirical methodology, relying on the cross-sectional dimension of the data rather than on parametric time-series restrictions, and thus provide independent evidence that the reversal of UIP violations is indeed a robust feature of the data. Second, I show that there is a clear cross-sectional relationship between the cyclicality in UIP violations and the monetary policy stance.

The theoretical mechanism itself is novel to the UIP literature, which largely turns to one of two explanations: time-varying risk (e.g. Bekaert (1996), Alvarez et al. (2009), Verdelhan (2010), Gabaix and Maggiori (2015), Farhi and Gabaix (2015), Bansal and Shaliastovich (2012), Colacito and Croce (2013), Hassan (2013)), and deviations from rational expectations (Gourinchas and Tornell (2004), Bacchetta and Van Wincoop (2010), Burnside et al. (2011), Ilut (2012)). Instead, I explore time-varying convenience yield differentials, and specifically focus on analyzing the changing nature of UIP violations, whereas the literature has concentrated on the classic short horizon puzzle.[4] Another interesting differentiating feature of the model is the explicit role for monetary policy, which is well documented in the data.

Two more closely related papers are Engel (2016) and Itskhoki and Mukhin (2016), who single out exogenous volatile, but transitory, shocks to liquidity as potential solutions to exchange rate puzzles. This paper shares their insight that liquidity values are important, but instead I develop a framework where the convenience yield itself is endogenous, and the changing nature of the puzzle is a result of the endogenous interaction between monetary and fiscal policies. Moreover, an especially interesting avenue for future research is combining my convenience yield mechanism, which is very successful at generating the documented non-linear, low-frequency dynamics of UIP violations, with high-frequency risk-premium fluctuations that could help explain the high volatility of short-term currency returns.[5]

A number of papers have quantified convenience yields in the data and documented their important role in the determination of equilibrium bond prices (e.g. Fontaine and Garcia (2012), Krishnamurthy and Vissing-Jorgensen (2012), Smith (2012), Greenwood and Vayanos (2014)). A related theoretical literature has explored bond convenience yields as a possible explanation for closed economy asset pricing puzzles such as the equity risk-premium, the low risk-free rate and the term premium (e.g. Bansal and Coleman (1996), Lagos (2010), Bansal et al. (2011) respectively). I extend the theoretical analysis of convenience yields by introducing them to an open economy setting, and studying their implications about

[4] The model can also rationalize the Hassan and Mano (2015) findings that a significant portion of carry trade profits is not due to time-variation in excess returns, but rather due to persistent differences in excess returns across currencies via steady state differences in convenience yields.

[5] Convenience yields could also be acting as omitted variables in attempts to relate traditional risk factors to currency returns, which have only had mixed results (e.g Burnside (2011), Menkhoff et al. (2012b)).

exchange rate determination. I also provide new empirical results showing that convenience yields appear to be important drivers of exchange rates in the data.

The paper is organized as follows. Section 2 establishes the motivating empirical facts, and Section 3 introduces the idea of convenience yields. Section 4 lays out and analyzes the analytical model, while Section 5 presents the quantitative model. Sections 6 and 7 provide direct empirical evidence in support of the mechanism, and Section 8 concludes.

2 Empirical Evidence

I begin by presenting evidence on the failure of UIP at different horizons. I use daily data on forward and spot exchange rate contracts for 18 advanced OECD countries for the period 1976:M1 - 2013:M6. The data comes from Datastream and is described in detail in Online Appendix A.[6]

2.1 UIP Violations at Short and Long horizons

Up to a first order approximation, standard international models imply that the rates on return on risk-free assets across countries are equalized. This condition is known as Uncovered Interest Parity (UIP), and in particular requires that the expected exchange rate depreciation offsets any potential gains from differences in interest rates so that

$$E_t(s_{t+1} - s_t) = i_t - i_t^*,$$

where s_t is the log exchange rate in terms of home currency per one unit of foreign currency, i_t and i_t^* are the home and foreign interest rates. This condition puts important restrictions on the joint dynamics of exchange rates and interest rates, and as a result plays a crucial role in exchange rate determination in standard models. Its empirical failure, however, is one of the best established facts in international finance, with a large literature expanding on the seminal contributions by Bilson (1981) and Fama (1984).[7]

The UIP condition is traditionally tested by examining whether the excess return of foreign bonds over home bonds, i.e. the 'excess currency return', is forecastable. I denote

[6] The 18 currencies are for Australia, Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland and the UK.

[7] See also Canova (1991), Canova and Ito (1991), Bekaert and Hodrick (1992), Backus et al. (1993), Canova and Marrinan (1993), Cheng (1993), Hai et al. (1997), Bekaert (1995), Burnside (2013). Lewis (1995), and Engel (1996, 2013) provide excellent surveys. A related finding is the high profitability of the carry trade, an investment strategy that is long high-interest rate currencies and short low-interest rate ones, and should yield zero average return under UIP – see Lustig and Verdelhan (2007), Burnside et al. (2008), Brunnermeier et al. (2008), Burnside et al. (2010), Lustig et al. (2011), Menkhoff et al. (2012a)

the one period (log) excess return from time t to $t+1$ as λ_{t+1}:

$$\lambda_{t+1} \equiv s_{t+1} - s_t + i_t^* - i_t.$$

The UIP condition requires $E_t(\lambda_{t+1}) = 0$, and hence $\text{Cov}(\lambda_{t+1}, X_t) = 0$ for any variable X_t in the time t information set. The vast majority of the literature focuses on some version of the original regression specification estimated by Fama (1984):

$$\lambda_{t+1} = \alpha_0 + \beta_1(i_t - i_t^*) + \varepsilon_{t+1}, \tag{1}$$

where typically the 'home' currency is the USD and i_t is the US interest rate. Under the null of UIP, $\beta_1 = 0$ so that the average excess return is not forecastable by current interest rates. To the contrary, numerous papers find that $\beta_1 < 0$, signifying that higher interest rates are associated with higher excess returns. This time variation in excess currency returns is a major challenge to standard models and $\beta < 0$ has traditionally defined the 'UIP Puzzle'.

However, recent work by Engel (2016) shows that this is not the whole story. Primarily focusing on real exchange rates and interest rates, he finds that while high interest rate differentials are associated with an increase in domestic currency excess returns in the short-run, they are in fact associated with a significant decrease in excess returns at longer horizons. Thus, in addition to the classic anomaly of insufficient depreciation at short horizons, it appears that high interest rate currencies tend to depreciate *too much* at longer horizons.

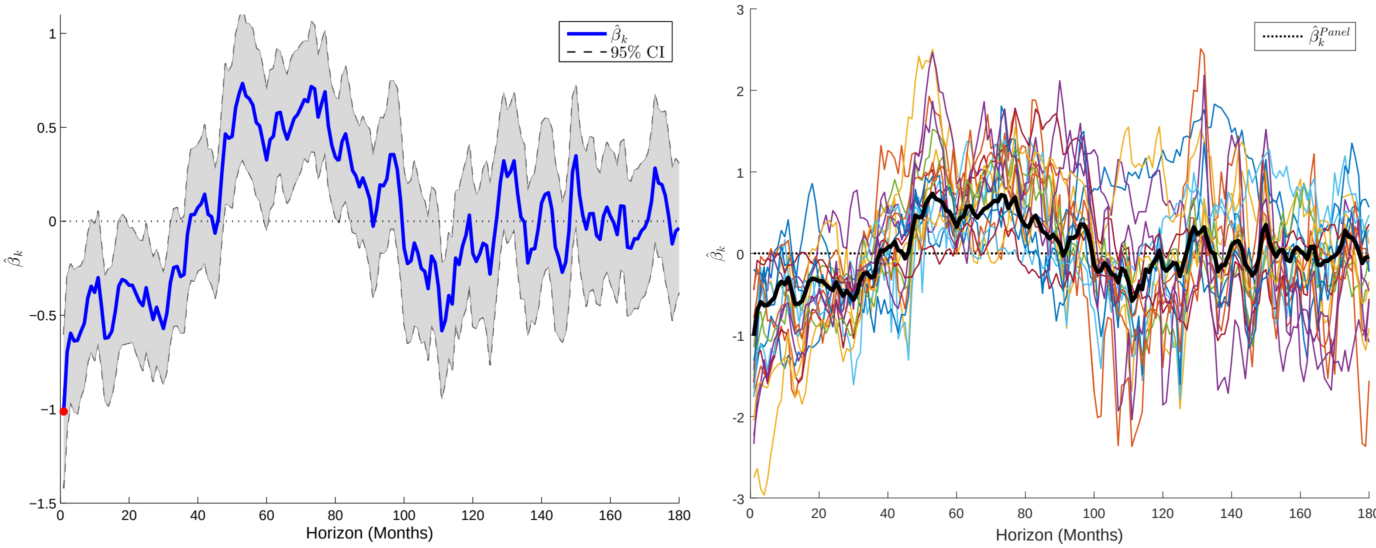
To capture both the short and long horizon anomalies, I generalize the standard UIP test to an arbitrary k-period ahead horizon. Applying the law of iterated expectations, it follows that for any $k > 0$

$$E_t(\lambda_{t+k}) = 0.$$

In essence, UIP implies that *any* future one-period excess return is unforecastable, not just the one-step ahead return, and this provides us with a series of testable conditions indexed by the horizon k. In contrast to Engel (2016), I work exclusively with nominal data and do not restrict the time-series dynamics to follow a VAR. Instead, I estimate

$$\lambda_{j,t+k} = \alpha_{j,k} + \beta_k(i_t - i_{j,t}^*) + \varepsilon_{j,t+k} \tag{2}$$

as a series of k separate panel regressions with fixed effects, where j indexes the currency and k the horizon in months. Thus, the left-hand side variable, $\lambda_{j,t+k}$, is the one-month excess return on the j-th currency from period $t+k-1$ to $t+k$. Note that the maturity of the investment is held constant at one month for all k, and only the forecasting horizon changes. In particular, for $k = 1$ we are back to the original Fama regression in eq. (1), for $k = 2$



(a) Panel estimates

(b) Currency-by-currency estimates

Figure 1: UIP Regression at horizons from 1 to 180 months

the left-hand side is the one-month excess return between periods $t+1$ and $t+2$, and so on. I focus on discussing the estimates from the panel regression, however as shown below, the currency-by-currency estimates are remarkably similar.

The left panel of Figure 1 plots the estimated coefficients $\hat{\beta}_k$ with the horizon k, in months, on the X-axis. The solid blue line plots the point estimates and the shaded region represents the 95% confidence intervals around each estimate, computed with Driscoll and Kraay (1998) standard errors that correct for heteroskedasticity, serial correlation and cross-equation correlation. The red dot on the plot is the point estimate of the classic UIP regression that looks just one month into the future.

The plot shows three important results. First, the coefficients are negative and statistically significant at horizons of up to 36 months. This corresponds to the common finding that following an increase in the interest rate differential, currencies fail to depreciate sufficiently to offset it and hence earn high excess returns – this is the classic 'UIP Puzzle'. However, the coefficients change sign at longer horizons, and are actually *positive* and statistically significant at horizons between 48 and 84 months. This signifies that high interest rates today forecast significantly *lower* excess returns at horizons of 4 to 7 years in the future, thus indicating a persistent *excess* currency depreciation at those horizons. This effect is the same order of magnitude as the classic short-horizon UIP puzzle, but runs in the opposite direction to it. And third, after a brief period at horizons of 100 to 120 months, the coefficients appear to converge to zero in the long-run. Overall, the UIP violations follow a clear, cyclical pattern, where they are negative at short horizons, positive at medium horizons, and gradually disappear in the long-run.

The right panel of Figure 1 plots the currency-by-currency estimates, and shows that the cyclical pattern is a remarkably consistent feature of all 18 currencies. This is an interesting result in of itself, and is also what allows me to obtain high statistical power, without having to impose parametric restrictions on the time-series dynamics.

The main takeaway from these results is that the nature of UIP violations changes with the horizon – the short-horizon violations are characterized by negative coefficients and the longer horizon violations by positive ones. The difference is not so much in the magnitude of the violations, which is roughly the same, but in their direction. At short horizons, exchange rates fail to depreciate sufficiently to fully offset the interest rate differential, while at longer horizons we have the opposite puzzling behavior, as exchange rates in fact depreciate *too much*. Thus, while UIP is violated at both short and long horizons, the fundamental nature of the violations changes with the horizon, suggesting that the excess currency returns have more complicated, cyclical dynamics than commonly thought.

The results bolster the initial findings of Engel (2016) and show that the changing nature of UIP violations are indeed a robust empirical phenomenon. In contrast to that paper, I focus on nominal exchange rates and interest rates and use a different empirical methodology that relies on the cross-sectional variation in the data, instead of imposing parametric restrictions on dynamics through a VAR system. Given our different approaches it is re-assuring to obtain such complementary results, and further establishes the basic finding of the reversal in UIP violations as a robust feature of exchange rate dynamics.

2.2 Implications for Exchange Rate Behavior

Next, I delve deeper in the underlying exchange rate behavior, and show that the above results arise because the exchange rate exhibits a particular type of 'delayed overshooting' with eventual *excess* depreciation. I find that following an increase in the interest differential, there is a sustained exchange rate appreciation that generates the short-horizon negative UIP violations, and a subsequent excess depreciation at longer horizons that drives the positive UIP violations. Interestingly, the eventual depreciation more than offsets the initial appreciation, and in the long-run the exchange rate converges to the path implied by UIP.

To show this, I compare the actual response of the exchange rate to a change in the interest rate differential, to the counter-factual path under UIP. To avoid non-stationarity issues, I work with the cumulative change of the nominal exchange rate, $s_{t+k} - s_t$, and study the response *relative* to today's value. I estimate the impulse response function (IRF) using the Jorda (2005) method of local projections, which amounts to separately projecting each

k-periods cumulative exchange rate change on the current interest rate differential

$$\text{Proj}(s_{t+k} - s_t | i_t - i_t^*) = \gamma_k (i_t - i_t^*).$$

The sequence $\{\gamma_k\}$ forms an estimate of the IRF. The method of local projections is especially well suited for estimating long-run responses because of its flexible nature – there are no restrictions on the dynamics from period to period, as the response at each horizon is estimated via a separate projection. The coefficients γ_k are estimated through a series of fixed-effects panel regressions as in Section 2.1.

To obtain the UIP counter-factual, re-arrange the equation $\lambda_{t+1} = s_{t+1} - s_t + i_t^* - i_t$ to isolate the exchange rate change and sum forward

$$s_{t+k} - s_t = \sum_{h=1}^{k} (i_{t+h-1} - i_{t+h-1}^*) + \sum_{h=1}^{k} \lambda_{t+h}. \tag{3}$$

This expresses the exchange rate as the sum of future interest rate differentials and excess returns. Letting ρ_k be the k-th autocorrelation of the interest rate differential, and projecting both sides of (3) onto $i_t - i_t^*$ leads us to:

$$\gamma_k = \underbrace{\sum_{h=1}^{k} \beta_h}_{\text{Sum of UIP Violations}} + \underbrace{\sum_{h=0}^{k-1} \rho_h}_{\text{Sum of Expected Int Diffs}}$$

Under UIP, the excess returns are zero ($\beta_h = 0$) and hence the counter-factual path of the exchange rate under UIP depends only on the dynamics of the interest rate differentials:

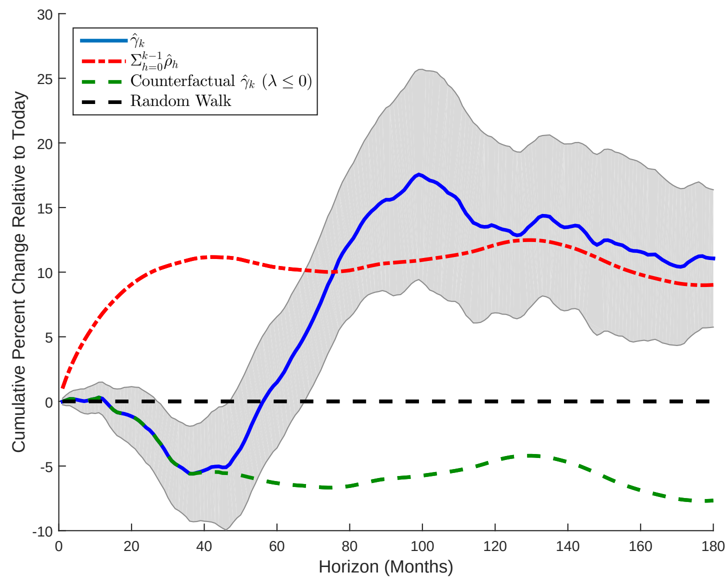
$$\gamma_k^{UIP} = \sum_{h=0}^{k-1} \rho_h,$$

I estimate the needed ρ_k coefficients with a similar fixed-effects panel regressions.

Figure 2 shows the results. The blue line plots the actual IRF, $\hat{\gamma}_k$, with its 95% confidence interval as the shaded area around it, and the red dash-dot line plots the UIP counter-factual. The initial diverging movements of the blue and red lines generates the classic UIP puzzle. An increase in the interest rate generates a persistent rise in the interest differential, and hence UIP predicts that in response the exchange rate will experience a sustained depreciation that offsets this differential. On the contrary, however, the exchange rate fails to depreciate and in fact even appreciates at horizons of up to 36 months. Thus, the exchange rate response does not close profit opportunities but rather enhances them, leading to high interest rates forecasting high excess currency returns in the short-run.

The initial appreciation is not the whole story, however, as the exchange rate eventually reverses course, and experiences a sharp depreciation at horizons of four to seven years. This depreciation is in *excess* of UIP, and thus leads to a fall in the excess currency return, and generates the corresponding positive UIP violations at longer horizons. These results do not simply restate the well known "delayed overshooting" property of exchange rate dynamics (e.g. Eichenbaum and Evans (1995)), but add to our understanding of it by highlighting that the eventual depreciation is in excess of UIP. Hence the exchange rate does not simply eventually start behaving as UIP would imply, but in fact over-reacts in the other direction.

Figure 2: Exchange Rate IRF



Interestingly, the excess depreciation is strong enough to fully offset the initial appreciation and to catch up the exchange rate with the UIP-implied path, making it seem like the long-run exchange rate behavior is consistent with UIP, even though UIP is violated at every step of the way. This provides an interesting new interpretation of the findings of Flood and Taylor (1996), Chinn (2006) and others, who show that long-term investments (5+ years) exhibit significantly smaller, often insignificant UIP violations, suggesting that UIP might hold well in the long-run. Instead, my results imply that long-run investments held to maturity do not display significant excess returns because the initial short-run gains are offset by the excess depreciation at longer horizons. Thus, UIP is violated in both the short and the long-run, but in such a way that the total sum of violations is roughly zero.

Lastly, the fact that the nominal exchange rate converges to the UIP counter-factual in the long-run is in contrast to the dynamics of the real exchange rate estimated in Engel

(2016). He finds that the eventual real excess depreciation is significantly stronger than the initial appreciation, not equal to it as I find for the nominal exchange rate. This suggests that there are meaningful differences in the nominal and real exchange rate dynamics, at least at medium to long horizons.

To further illustrate the significance of the changing sign of UIP violations, the green dashed line in Figure 2 plots a counter-factual exchange rate path ($\gamma_k^{\lambda \leq 0}$) obtained by setting $\beta_k = 0$ for all $k \geq 36$. This simulates exchange rate behavior in a world where UIP violations are always negative and do not change sign. The main difference is that the exchange rate fails to turn around after the initial appreciation, and this leads to a counter-factual long-run exchange rate behavior. Thus, in order to obtain the appropriate, non-monotonic exchange rate dynamics, a model needs to account for the cyclical pattern in UIP deviations.

2.3 UIP Violation Reversals and Monetary Policy

Next, I explore the cross-sectional relationship between UIP reversals and monetary policy. I am motivated by the results of Bansal and Dahlquist (2000), who find that the the classic UIP Puzzle is much less pronounced, for countries with high or volatile inflation. I extend their work by showing that there is a similarly strong cross-sectional link between monetary policy and the newer evidence of reversals in the UIP violations at longer horizons.

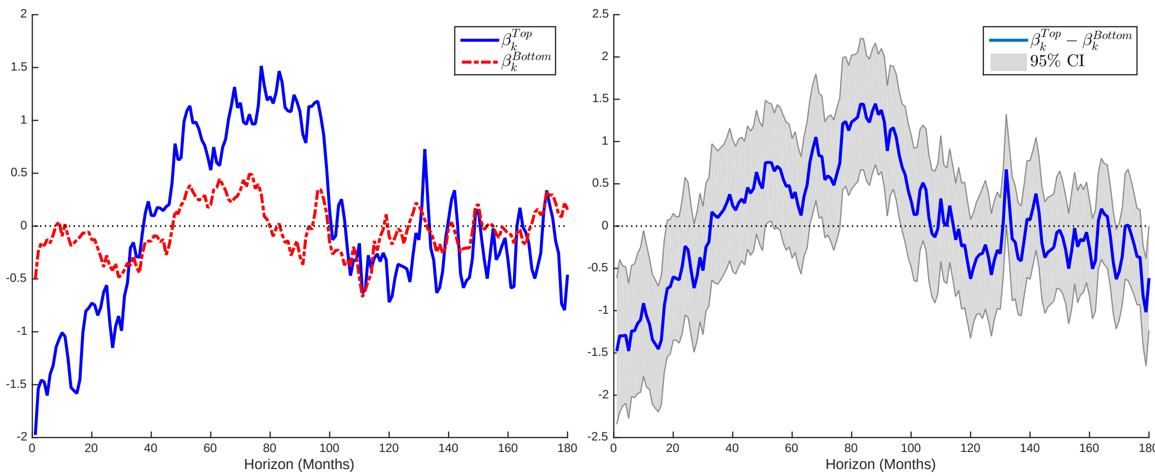
For completeness, I consider four different proxies for the monetary stance of a country. In addition to the two proxies used in Bansal and Dahlquist (2000), average inflation and the standard deviation of inflation, I use the Central Bank Independence Index (CBI) of Grilli et al. (1991) (updated with recent data by Arnone et al. (2007)), and the degree of capital controls, as measured by the Chinn and Ito (2006) index.[8] Since the proxies are generally only available at a low frequency, I focus on exploiting the cross-sectional dimension of the data. For each currency, I compute the corresponding average value for each proxy (e.g. average CBI for the UK over 1976-2013 and etc.), and then for each proxy I sort the currencies into three bins – high, middle and low. Finally, I find the intersection of all the top bins, which yields three countries (Germany (DEM), the Netherlands (NLG), and Switzerland (CHF)) that score in the top third in all measures of monetary policy independence. And similarly obtain the intersection of the bottom bins, which yields (Ireland (IEP), Italy (ITL), Spain (ESP), Portugal (PTL)). Then I re-estimate the series of UIP regressions at different horizons, eq. (2), for both sets separately and compare the results.

Figure 3 plots the estimates and shows a remarkably consistent message. In panel a) we see that currencies with high monetary independence display a much more pronounced

[8] Capital controls are commonly used as a *de facto* measure of CB independence – see for example Alesina and Tabellini (1989), Drazen (1989), Grilli and Milesi-Ferretti (1995), and Bai and Wei (2000)

Figure 3: UIP Violations and Monetary Policy

(a) Top vs Bottom Third (b) $\beta_k^{Top} - \beta_k^{Bottom}$



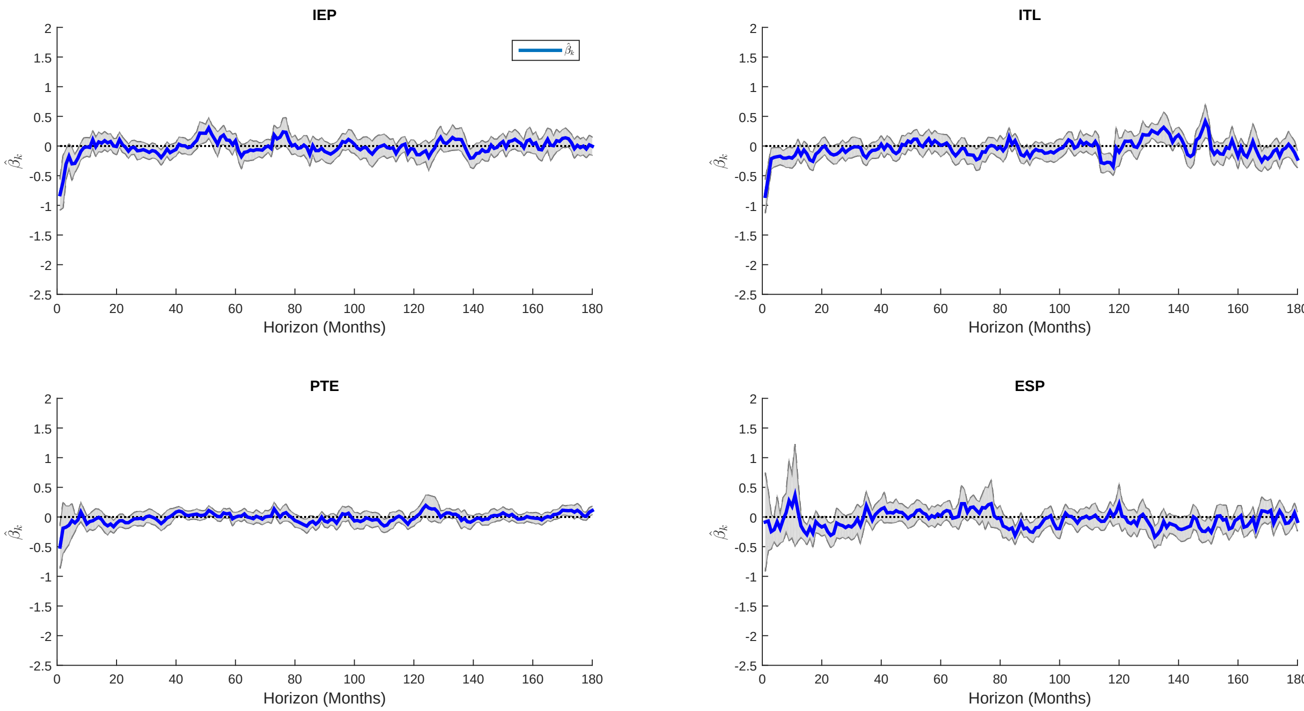
evidence of cyclicality in UIP violations, and generally exhibit a larger magnitude of UIP violations at all horizons. Panel b) shows that the difference between the two estimates, $\beta_k^{Top} - \beta_k^{Bottom}$, is in fact statistically significant (at the 5% level). Thus, currencies with a more independent monetary policy do not only display larger UIP violations at short-horizons, but also stronger evidence of a reversal in their direction at longer horizons.[9]

Lastly, note that all currency pairs in the above results are quoted against the USD. However, since the US scores high in all four proxies, one leg of each currency pair displays strongly independent monetary policy throughout the whole sample. It is interesting to also consider results where the base currency has low monetary independence. To do so, I use the set of currencies that are in the bottom bin according to all proxies (IEP, ITL, ESP, PTE) as alternative base currencies, and construct four different sets of currency pairs (e.g. ITL-AUD, ITL-ATS, ...). This gives me four data sets of 18 currencies each, that I then use to re-estimate the initial set of regressions in eq. (2).

The results are plotted in Figure 4 and are quite striking – in all four plots the UIP violations exhibit virtually no evidence of a reversal. Thus, it appears that the cyclicality in UIP violations is associated with strong and independent monetary policy. This suggests that monetary policy, and also fiscal policy to the extent that the interaction of the two determine equilibrium inflation, play an important role in this phenomenon. With this in mind, in the rest of the paper I turn attention to developing a model with an explicit role for monetary policy that can generate UIP reversals as in the data.

[9] The results also hold individually for each proxy.

Figure 4: UIP Regressions, 1 to 180 months



3 Time-Varying Convenience Yields and Exchange Rates

UIP relies on three key assumptions: constant risk-premia, rational expectations and that financial returns are the only benefit to holding bonds. Deviations from the first two have been extensively analyzed in the previous literature, and instead this paper focuses on relaxing the third assumption by introducing a non-pecuniary benefit to holding bonds.

This is motivated by the literature documenting a significant, time-varying “convenience yield” component in government bond yields (Reinhart et al. (2000), Longstaff (2004), Krishnamurthy and Vissing-Jorgensen (2012), Greenwood and Vayanos (2014)). The convenience yield is the amount of interest investors are willing to forego in exchange for the non-pecuniary benefits of owning high-quality debt. Those benefits arise from the high safety and liquidity of risk-free debt, which makes it a good substitute for money, a special asset that investors are willing to hold at zero interest rate. For example, Treasuries serve an important role as collateral in facilitating complex financial transactions, back deposits, and often even act as direct means of payment between financial institutions. In addition, there is a demand for safe assets from mutual funds and/or insurance companies mandated to invest in certain class of assets, government units such as the Social Security Fund, and long-horizon investors. Hence, bonds provide many of the special features of money as medium of exchange and store of value, and as a result share in some of its holding benefits.

In an international context, the convenience yield differential between the short-term debt of two countries, $\hat{\Psi}_t$, acts as a wedge in the Euler equation, such that up to first-order

$$E_t(s_{t+1} - s_t + i_t^* - i_t) = \hat{\Psi}_t. \tag{4}$$

In other words, investors would balance not only the expected relative financial return on the two bonds, but also the differences in their liquidity values. Thus, in equilibrium currency returns adjust to offset the convenience yield differential – when the home bond convenience yield is high, investors require a higher financial return on the foreign bond as compensation, which gives rise to time-variation in excess currency returns, and violates UIP.

This is a wedge that has not been studied previously as a possible explanation of the UIP puzzle, but is a potentially important force. Empirical estimates of the average convenience yield on US Treasuries, for example, range between 75 and 166 basis points, and estimates of the standard deviation range between 45 and 115 bp (see Krishnamurthy and Vissing-Jorgensen (2012), Krishnamurthy (2002), Longstaff et al. (2005)). It is a large and volatile component that could have a significant impact on estimated UIP violations.[10].

Moreover, recent work has shown that while exchange rates do not appear to offset the interest rate differential of high-quality short-term debt assets, they do respond to expected return differentials of other, less special assets. In particular, Lustig et al. (2015) study the returns of a currency trading strategy that takes short-term (1 month) positions in long-term bonds. They find that this version of the carry trade earns surprisingly low returns, that are in fact roughly zero in the case of bonds with three year maturity or longer. Furthermore, in separate time-series regressions tests they also find that the expected returns on this type of short-term investment in long-term bonds is equalized across currencies. On the other hand, Cappiello and De Santis (2007), Hau and Rey (2006), and Curcuru et al. (2014) test whether differences in expected monthly equity returns across countries are offset by exchange rate movements, and find that indeed they are, in contrast to the typical result of UIP tests. Thus, it appears that excess currency returns are non-zero only when transacting in assets close to money, suggesting that convenience yields could play an integral role.

To explore this hypothesis further, I develop a model with endogenous fluctuations in equilibrium convenience yields and an explicit role for monetary policy, and then test its key implications in the data. After presenting the model, I also discuss how it relates to recent

[10] Also a number of papers show that convenience yields can help account for different closed economy asset pricing puzzles, such as the low equilibrium risk-free rate, the equity risk premium, and the term premium (Bansal and Coleman (1996), Bansal et al. (2011), Lagos (2010), Acharya and Viswanathan (2011))

work by Engel (2016) and Itskhoki and Mukhin (2016), which argue that *exogenous* shocks to liquidity may hold promise to understanding exchange rate fluctuations. Lastly, the effects studied here are first-order, and thus combining this model with traditional time-varying risk-premia mechanisms would serve to reinforce both.

4 Analytical Model

I start by present an intentionally stylized version of the model that allows for analytical results and a clean illustration of the main mechanism. In the next section, I relax the simplifying assumptions made here, set the mechanism in a two country general equilibrium model, and show that all the insights from this section transfer fully.

In the analytical model, there are two countries, a large home country and a small foreign country that is negligible in world equilibrium.[11] The household faces incomplete financial markets, where it can only trade home and foreign nominal bonds. The bonds are supplied by the respective governments, which set monetary policy via a Taylor rule and finance a fixed level of expenditures by levying lump-sum taxes and issuing nominal debt.

The key innovation in this otherwise standard framework is that in addition to the interest payment, bonds also offer a non-pecuniary, convenience benefit. I follow the recent bond convenience yield literature and adopt a "bonds-in-the-utility" approach that imposes minimal restrictions on the general form of the preference for liquidity. However, the results also hold under more specific frameworks, such as cash-in-advance or search theory of money. Lastly, the analytical model studies the limiting case of a cashless economy.

4.1 The Household

The household is infinitely lived and maximizes the expected sum of future utility,

$$\sum_{k=0}^{\infty} E_t \beta^k u(c_{t+k}, b_{h,t+k}, b_{f,t+k})$$

where $u(\cdot)$ is concave, c_t is consumption, and b_{ht} and b_{ft} are the real holdings of home and foreign bonds respectively. We do not need to specify preferences any further, except for the assumption that home and foreign bonds are not perfect substitutes, so that

$$|u_{b_h b_h}(.)| > |u_{b_h b_f}(\cdot)|$$

[11] This setup follows Bacchetta and van Wincoop (2006). It can be thought of as a partial equilibrium two-country model, where the foreign country is taken as exogenous. Again, this assumption is relaxed later.

where $u_{xx}(.)$ is the second partial derivative of the utility. Intuitively, this condition states that the marginal benefit of home bonds is more sensitive to acquiring an extra unit of home bonds, than to acquiring an extra unit of foreign bonds – i.e. home bonds tend to be more useful than foreign ones. A way to think about this is that the household consumes both home and foreign goods, but with a bias towards the home good, and hence both home and foreign liquidity is useful, but home liquidity more so.[12]

The household faces the following budget constraint at date t

$$c_t + b_{ht} + b_{ft} = y - \tau_t + b_{h,t-1}\frac{(1+i_{t-1})}{\Pi_t} + b_{f,t-1}\frac{(1+i^*_{t-1})}{\Pi_t}\frac{S_t}{S_{t-1}}$$

where y is a constant endowment of the consumption good, τ_t are real lump-sum taxes, Π_t is the gross inflation rate, i_t and i^*_t are the domestic and foreign nominal interest rates, and S_t is the nominal exchange rate. This leads to the following Euler equations:

$$1 = \beta E_t\left(\frac{u_c(c_{t+1}, b_{h,t+1}, b_{f,t+1})}{u_c(c_t, b_{ht}, b_{ft})}\frac{1+i_t}{\Pi_{t+1}}\right) + \frac{u_{b_h}(c_t, b_{ht}, b_{ft})}{u_c(c_t, b_{ht}, b_{ft})}$$

$$1 = \beta E_t\left(\frac{u_c(c_{t+1}, b_{h,t+1}, b_{f,t+1})}{u_c(c_t, b_{ht}, b_{ft})}\frac{1+i^*_t}{\Pi_{t+1}}\frac{S_{t+1}}{S_t}\right) + \frac{u_{b_f}(c_t, b_{ht}, b_{ft})}{u_c(c_t, b_{ht}, b_{ft})}$$

The Eulers equate the real cost of an extra unit of investment in bonds to the discounted, expected payoff. The cost is the unit of foregone consumption today and the payoffs are composed of both financial returns and a convenience benefits. For example, the top equation shows that an additional unit of home bonds offers a financial return of $\frac{1+i_t}{\Pi_{t+1}}$ (appropriately discounted), plus a convenience benefit of $\frac{u_{b_h}(c_t,b_{ht},b_{ft})}{u_c(c_t,b_{ht},b_{ft})}$ (in terms of consumption). For future reference, I define the marginal convenience benefits of home and foreign bonds as

$$\Psi^H_t \equiv \frac{u_{b_h}(c_t, b_{ht}, b_{ft})}{u_c(c_t, b_{ht}, b_{ft})}\ ;\ \Psi^F_t \equiv \frac{u_{b_f}(c_t, b_{ht}, b_{ft})}{u_c(c_t, b_{ht}, b_{ft})}$$

Note that these are endogenous equilibrium objects – they depend on equilibrium consumption, and home and foreign bond holdings.

[12] In the sake of generality, I analyze the case where both foreign and home bonds potentially enter the utility, however, this is not necessary for any of the results – i.e. we could set $u_{b_f} = 0$ without losing much.

4.2 The Government

The government sets monetary policy according to a standard Taylor rule

$$\frac{(1+i_t)}{1+i} = (\frac{\Pi_t}{\Pi})^{\phi_\pi} e^{v_t}$$

where v_t is white noise. On the fiscal side, it faces a constant level of real expenditures g and the budget constraint

$$b_t^G + \tau_t = \frac{(1+i_{t-1})}{\Pi_t} b_{t-1}^G + g$$

where b_t^G is real government debt. I follow the literature on the interaction of monetary and fiscal policy and assume that the lump-sum taxes are set according to the linear rule[13]

$$\tau_t = \rho_\tau \tau_{t-1} + (1-\rho_\tau)\kappa_b b_{t-1}^G,$$

where $\rho_\tau \in [0,1)$ is a smoothing parameter and $\kappa_b \geq 0$ controls how strongly taxes respond to debt levels. The rule models the general idea that the government adjusts taxes to stay solvent, but does so gradually. This policy framework is not meant to capture optimal policy, but rather model government behavior in a tractable and, yet, empirically relevant way.

4.3 Steady State

At the zero-inflation steady state, the Euler equations for domestic bonds in the home and the foreign country, respectively, reduce to

$$1+i = \frac{1}{\beta}(1-\Psi^H) \quad \text{and} \quad 1+i^* = \frac{1}{\beta}(1-\Psi^F),$$

where $\frac{1}{\beta}\Psi^H$ and $\frac{1}{\beta}\Psi^F$ are the steady state convenience yields. A higher steady state convenience yield is associated with a lower i, because domestic investors require lower financial compensation for holding home debt when its convenience value is high.

The interest rate differential and the steady state excess currency returns are given by

$$i - i^* = \frac{1}{\beta}(\Psi^F - \Psi^H)$$

$$(1+i^*)\frac{S'}{S} - (1+i) = \underbrace{\frac{1}{\beta}(\Psi^H - \Psi^F)}_{\text{Convenience Yield Differential}}$$

[13] See for example Leeper (1991), Chung et al. (2007), Davig and Leeper (2007). Also, fiscal policy can instead be implemented through a rule on expenditures (g_t), without changing the results.

Thus, if there are cross-sectional differences in the steady state convenience values of assets denominated in different currencies, this will drive a corresponding difference in their steady state interest rates as well. In addition, differences in the convenience yields will also lead to a non-zero steady state excess currency return. When the home convenience yield is higher than the foreign one, the foreign currency will be compensated through a positive excess return, in order to keep investors indifferent between home and foreign bonds.

Hence, the model can explain the Hassan and Mano (2015) evidence that a big portion of carry trade returns are due to persistent cross-sectional differences in currencies and unconditional premia, and not time-variation in conditional premia. For example, the model would imply that part of the reason why the Japanese yen is consistently a funding currency and the Australian dollar is consistently an investment currency, is because the Japanese yen is a major international reserve currency while the Australian dollar is not. As such, the yen earns a higher convenience yield on average, and thus has a lower interest rate and negative excess returns versus the Australian dollar.

Thinking about the drivers of the unconditional premia of carry trades is an interesting question, but is distinct from the primary motivation of this paper – the cyclical nature of UIP violations. To understand the UIP regression evidence, and its changing nature at different horizons, one needs to understand the equilibrium dynamics of the conditional excess currency returns. To this end, in this paper I focus on the symmetric steady state where $\Psi^H = \Psi^F$ in order to isolate the effect of the time-variation in the convenience yield.

4.4 Currency Returns and UIP Violations

Log-linearizing the home bonds Euler equation around the symmetric steady state yields:

$$\hat{i}_t - E_t(\hat{\pi}_{t+1}) + \frac{\Psi^H}{\beta(1+i)}\hat{\Psi}_t^H = -E_t(\hat{M}_{t+1}) \tag{5}$$

where $M_{t+1} = \frac{u_{c,t+1}}{u_{c,t}}$ is the MRS, and hats denote log-deviations from steady state. The left-hand side is the real return on home government debt – the real interest rate plus the convenience yield. The right-hand side is the negative of the MRS, which is equal to the return of an asset with no convenience benefits, and hence the convenience yield is the amount of interest agents are willing to forgo in exchange for the convenience benefits. Naturally, there is a negative relationship between the convenience yield and the interest rate – the higher the convenience yield, the lower the interest rate agents requires to hold home debt.

Log-linearizing the foreign bonds Euler leads to a similar condition, and combining the two, we obtain an expression for the equilibrium excess currency returns

$$E_t(\hat{s}_{t+1} - s_t + \hat{i}^*_t - \hat{i}_t) = \frac{\Psi^H}{\beta(1+i)}(\hat{\Psi}^H_t - \hat{\Psi}^F_t) \tag{6}$$

This shows that uncovered interest parity does not hold – there are predictable excess returns in equilibrium that arise as a compensation for differences in the convenience yields on home and foreign bonds. When the home bond's equilibrium convenience yield increases, the foreign bond is compensated with higher expected financial returns and vice versa. Without this convenience yield mechanism, there will be no UIP violations in the model.[14]

For simplicity, in the analytical model I assume that foreign monetary policy keeps interest rates fixed, which implies that the interest rate differential is given by

$$\hat{i}_t - \hat{i}^*_t = \hat{i}_t = E_t(\hat{\pi}_{t+1}) - E_t(\hat{M}_{t+1}) - \frac{\Psi^H}{\beta(1+i)}\hat{\Psi}^H_t \tag{7}$$

We can already see how the classic UIP puzzle relationship is a fundamental feature of the mechanism, due to the negative relationship between the interest rate and the domestic convenience yield. Equations (6) and (7) imply that periods when the home convenience yield is low are associated with a high interest rate differential, and high domestic excess currency return. Applying the law of iterated expectations to (6) results in

$$E_t(\hat{s}_{t+k+1} - s_t + \hat{i}^*_{t+k} - \hat{i}_{t+k}) = \frac{\Psi^H}{\beta(1+i)}E_t(\hat{\Psi}^H_{t+k} - \hat{\Psi}^F_{t+k}) \tag{8}$$

showing that future excess currency returns equal the future expected convenience yield differential. Hence the behavior of UIP violations at longer horizons depends on the equilibrium dynamics of the convenience yield differential, which I characterize next.

4.5 Equilibrium Dynamics

The foreign country is small and does not affect world markets, hence equilibrium in the goods market implies that the home household's consumption is constant over time:

$$c_t = c + g = y.$$

[14]Gabaix and Maggiori (2015) develop a model based on a different notion of liquidity, where financial intermediaries face borrowing constraints and have a limited ability to absorb global imbalances, which drives a time-varying currency risk-premium. The mechanism here is different, the excess currency returns are in compensation for differences in the liquidity *value* of home and foreign bonds, and are not related to risk.

The small size of the foreign country also implies that foreign bonds are in zero net supply, $b_{ft} = 0$, and that home agents must hold the whole supply of home bonds:

$$b_{ht} = b_t^G.$$

Thus, since equilibrium consumption and foreign bond holdings are constant, the equilibrium convenience yield dynamics are entirely determined by home government debt

$$\frac{\Psi^H}{\beta(1+i)}\hat{\Psi}_t^H = -\gamma_\Psi \hat{b}_{ht}^G \tag{9}$$

where $\gamma_\Psi > 0$. The convenience yield is decreasing in the amount of home bonds owned by the agent, as the preferences for liquidity exhibit diminishing marginal utility. Substituting (9) in the rest of the log-linearized equilibrium conditions, the model reduces to a system of four equations – the Euler equation for home bonds, the government budget constraint, the Taylor rule and the tax rule. This system of equations determines the equilibrium values of home debt, inflation, taxes and the interest rate.

The model admits two types of determinate stationary equilibria, and which one obtains depends on the interaction between monetary and fiscal policy. I use the standard terminology in the literature and call a policy 'active' when it is unconstrained by the government budget and can actively pursue its objective. And 'passive' when it needs to obey the equilibrium constraints imposed by the other policy authority, and passively adjusts the variable under its control, either interest rates or taxes, to keep the government solvent.

One type of equilibrium obtains under the combination of active monetary and passive fiscal policies, where the monetary authority reacts strongly to inflation ($\phi_\pi > 1$), while the fiscal authority adjust taxes to fully fund its debts. The other is its mirror image, where the fiscal authority is active and does not adjust taxes strongly, and deficits must be financed by the passive monetary authority ($\phi_\pi < 1$) which allows inflation to rise and inflate debt away as needed. Lemma 1 formally characterizes both.

LEMMA 1 (Existence and Uniqueness). *A determinate stationary equilibrium exists if and only if we have one of the following two policy combinations:*

(i) Active Monetary, Passive Fiscal policy: $\phi_\pi > 1$, $\kappa_b \in (\theta - \theta_2, \frac{1+\rho_\tau}{1-\rho_\tau}(\theta + \theta_2))$, $\rho_\tau \in [0, \frac{\theta_2}{\theta})$.

(ii) Passive Monetary, Active Fiscal policy: $\phi_\pi < 1$, $\kappa_b \notin (\theta - \theta_2, \frac{1+\rho_\tau}{1-\rho_\tau}(\theta + \theta_2))$, $\rho_\tau \in [0, 1)$.

where $\theta > \theta_2 \geq 1$, *with* $\theta = (1+i)(1 + \gamma_\Psi + \gamma_M)$, $\theta_2 = 1 + \gamma_M(1+i)$, $\gamma_\Psi > 0$, *and* $\gamma_M \geq 0$.

Proof. The key is that the system of equilibrium conditions can be reduced to two first-order difference equations, which can be solved analytically using standard techniques. The text sketches the proof and gives intuition, while the details are in the Online Appendix C.1. □

To gain some intuition about the result, notice that the equilibrium MRS is $E_t(\hat{M}_{t+1}) = \gamma_M(E_t(\hat{b}_{h,t+1}) - \hat{b}_{ht})$, and thus the Euler equation for home bonds yields

$$\hat{\pi}_t = \frac{1}{\phi_\pi}\left(E_t(\hat{\pi}_{t+1}) + (\gamma_\Psi + \gamma_M)\hat{b}_{ht} - \gamma_M E_t(\hat{b}_{h,t+1}) - v_t\right). \tag{10}$$

If monetary policy is active ($\phi_\pi > 1$) we can use equation (10) to solve 'forward' for inflation, and express it as a sum of expected future debt levels and the monetary policy shock v_t. We can then date the government budget constraint one period ahead, take conditional time t expectation, and use the home bond Euler equation and the tax rule to get:

$$E_t \underbrace{\begin{bmatrix} \hat{b}_{h,t+1} \\ \hat{\tau}_{t+1} \end{bmatrix}}_{=x_{t+1}} = \underbrace{\begin{bmatrix} \frac{\theta-(1-\rho_\tau)\kappa_b}{\theta_2} & -\frac{\tau}{b}\frac{\rho_\tau}{\theta_2} \\ (1-\rho_\tau)\kappa_b\frac{b}{\tau} & \rho_\tau \end{bmatrix}}_{=A} \underbrace{\begin{bmatrix} \hat{b}_{ht} \\ \hat{\tau}_t \end{bmatrix}}_{=x_t}. \tag{11}$$

When the fiscal authority is passive, taxes adjust sufficiently strongly to debt (i.e. $\kappa_b > \theta - \theta_2$) to ensure that it is stationary and as a result, the eigenvalues of A are inside the unit circle. We can then use (11) to solve for $E_t(\hat{b}_{t+k})$ for any $k \geq 1$ and substitute it in the expression for inflation. Finally, use the resulting solutions for inflation and the interest rate rule to eliminate them both from the budget constraint, and combine with the tax rule to obtain a system of two equations in debt and taxes that we can solve 'backward':

$$\begin{bmatrix} b_{ht} \\ \tau_t \end{bmatrix} = A \begin{bmatrix} b_{h,t-1} \\ \tau_{t-1} \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1+i}{\phi_\pi} \\ 0 \end{bmatrix}}_{=B} v_t. \tag{12}$$

On the other hand, if monetary policy is 'passive' and $\phi_\pi < 1$, we cannot solve for inflation forward from equation (10). However, if fiscal policy is 'active' and taxes do not adjust strongly to movements in debt, $\kappa_b < \theta - \theta_2$, A has one eigenvalue greater than unity, and hence we can solve (11) forward for $\hat{b}_{ht}$. We can then solve for inflation and taxes.

The resulting dynamics of the two types of equilibria have important similarities and differences. To understand them better, I turn to the Impulse Response Function (IRF) of government debt to the monetary shock v_t. The Wold decomposition of $\hat{b}_{ht}$ is

$$\hat{b}_t = e_1 B v_t + e_1 A B v_{t-1} + e_1 A^2 B v_{t-2} + \ldots,$$

where $e_1 = [0, 1]$. The sequence $a_{bk} = e_1 A^k B$ forms the IRF and determines the equilibrium dynamics, and I characterize it in two steps – Lemma 2 looks at the Active Monetary/Passive Fiscal policy mix and Lemma 3 treats the Passive Monetary/Active Fiscal case.[15]

LEMMA 2 (IRF: Active Monetary/Passive Fiscal). *Let* $\phi_\pi > 1$, $\kappa_b \in (\theta - \theta_2, \frac{\theta + (\theta_2 - 1)\rho_\tau}{1-\rho_\tau})$, *and define* $\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b+\theta_2-\theta)+\theta\theta_2 - 2\sqrt{\kappa_b\theta\theta_2(\kappa_b+\theta_2-\theta)}}{(\theta_2+\kappa_b)^2} > 0$. *Then,*

(i) If $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ *the matrix A in (12) has two real, positive eigenvalues, and thus the IRF is positive and declines to zero monotonically:*

$$a_{bk} > 0 \text{ for } k = 0, 1, 2, 3, \ldots$$

(ii) If $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{\theta_2}{\theta})$ *the matrix A in (12) has a pair of complex conjugate eigenvalues,* $\lambda = a \pm bi$, *and conjugate eigenvectors* $\vec{v}_k = [x \pm yi, 1]'$, *where* a, b, x, y *are real numbers and i is the imaginary unit. Thus, the IRF follows the dampened cosine wave:*

$$a_{bk} = |\lambda|^k \sqrt{1 + (\frac{x}{y})^2} \cos(k\zeta + \psi - \frac{\pi}{2}), \text{ for } k = 1, 2, 3, \ldots$$

where $\zeta = \arctan(\frac{b}{a})$, $\psi = \arctan(\frac{y}{x})$ *and* $a_{bk} > 0$ *for* $k \in \{0, 1\}$.

Proof. Intuition is given in the text, and details are in Online Appendix C.2. □

Lemma 2 shows that under active monetary policy, the dynamics of the system are governed by real roots as long as taxes are not too persistent, and by complex roots otherwise. In both cases, the initial impact of a contractionary monetary shock is to increase home debt, but the subsequent dynamics differ. In the case of real roots the IRF is always positive and converges to steady state without crossing it, while under complex roots the IRF is positive initially, but follows a cyclical cosine function and crosses steady state before converging.

Consider the dynamics under real roots first. A contractionary monetary shock lowers inflation, which has two effects – it raises the real value of outstanding government debt, and increases the debt servicing costs through higher real interest rates. The two effects combine to worsen the budget position and increase real debt on impact. In response, the fiscal authority raises taxes to combat the elevated debt level, and if $\rho_\tau < \underline{\rho}(\kappa_b)$, taxes are sufficiently responsive to bring debt back to steady state in a controlled, monotonic fashion.

In the case of complex roots, the behavior on impact is similar, with both debt and the interest rate rising after a positive monetary shock. The transition dynamics back to steady

[15] I focus on the case $\kappa_b \leq \frac{\theta + (\theta_2 - 1)\rho_\tau}{1-\rho_\tau}$, which ensures that debt and taxes are positively autocorrelated, which is the empirically relevant case.

state, however, are different. They are characterized by a cosine curve with a frequency of oscillation controlled by $\psi = \arctan(\frac{b}{a}) \in (0, \frac{\pi}{2})$, where a and b are the real and the imaginary parts of the complex roots. The larger ψ is, the higher the frequency of oscillations, but the upper bound of $\frac{\pi}{2}$ ensures that the IRF stays above steady state for at least two periods. Still, the cyclical nature of the cosine dynamics leads debt to fall below steady state before converging, and how long it takes for this first crossing to occur depends on the size of ψ.

This cyclical behavior arises when tax policy is adjusting relatively sluggishly, i.e. $\rho_\tau > \underline{\rho}(\kappa_b)$, and as such it is relatively unresponsive to current debt levels. Intuitively, with smoothing taxes are a function of discounted past debt levels ($\hat{\tau}_t \propto \sum_{k=0}^{\infty} \rho_\tau^k \hat{b}_{t-k-1}$), and as a result taxes remain high even as debt approaches steady state, because they are still responding to past high debt levels. In other words, the tax increases enacted to combat the initial rise in debt are long-lived, and their lasting effect eventually pushes debt below steady state, giving rise to the cyclical dynamics formalized by the cosine curve. Looking forward to the dynamics of UIP violations, we'll see that whether or not debt crosses steady state also determines whether the excess returns (and thus UIP violations) change direction.

Lemma 3 summarizes the dynamics of the model under a Passive Monetary/Active Fiscal policy mix. In this case, the dynamics of the system are always characterized by real roots, regardless of how sluggish the tax policy is. The intuition is that with a Passive Monetary policy stance the key debt repayment mechanism is inflation and not taxes. Inflation, however, adjusts quickly in equilibrium and hence stabilizes debt without implying cyclical dynamics, regardless of the tax policy. In fact, in this simple model we have the stronger result that debt is constant, i.e. inflation completely insulates it from monetary shocks.[16]

LEMMA 3 (IRF: Passive Monetary/Active Fiscal). *Let $\phi_\pi < 1$, $\kappa_b \in [0, \theta - \theta_2)$, $\rho_\tau \in [0, 1)$. Then, the system has two real, positive eigenvalues for all $\rho_\tau \in [0, 1)$, and thus the IRF does not cross steady state. Moreover, debt is in fact constant:*

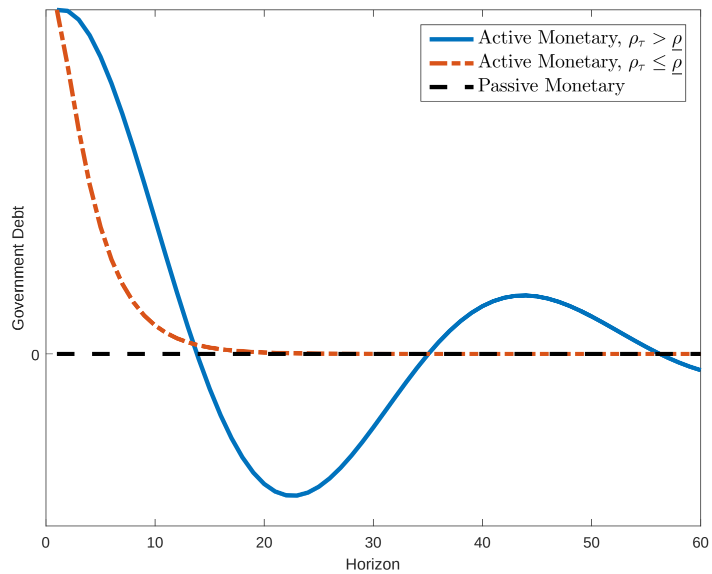
$$a_{bk} = 0 \text{ for } k = 0, 1, 2, 3, \ldots$$

Proof. See Online Appendix C.3. □

Figure 5 illustrates the types of dynamics we can obtain. Under active monetary policy, a contractionary shock increases debt on impact, and if taxes adjust quickly debt falls gradually back to steady-state, while with a sluggish tax rule it has cyclical dynamics. Under passive monetary policy, debt does not respond to monetary shocks, as inflation fully stabilizes it.

[16] The constant debt result is specific to monetary shocks – other shocks, e.g. fiscal shocks, move debt. The real eigenvalues result, however, is general and under passive monetary policy debt dynamics are not cyclical, regardless of the shock. We will see further evidence of this in the quantitative model.

Figure 5: Equilibrium Debt Dynamics



4.6 Main Analytical Results

Having determined equilibrium debt dynamics, we can now use equation (9) to obtain the equilibrium convenience yield dynamics as well, and thus also the equilibrium excess currency returns. In particular, plugging (9) into (8), the excess currency returns reduce to

$$E_t(\hat{\lambda}_{t+1}) = -\chi_b b^G_{ht}$$

where $\chi_b > 0$. As the stock of home debt increases, its convenience yield decreases and the equilibrium excess return on the home currency increases. We can similarly plug everything back into the home bonds Euler, to obtain

$$\hat{i}_t = E_t\hat{\pi}_{t+1} + (\gamma_\Psi + \gamma_M)\hat{b}_{ht} - \gamma_M E_t(\hat{b}_{h,t+1})$$

where $\gamma_\Psi > 0$ and $\gamma_M > 0$. Thus, debt pushes interest rates up, and excess foreign currency returns down, in line with the classic short-horizon UIP violations. Moreover, we can use the equilibrium debt dynamics we solved for in the previous section to fully characterize UIP violations, $\beta_k = \frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}^*_t)}{\text{Var}(\hat{i}_t - \hat{i}^*_t}$, at any horizon k.

Naturally, the profile of UIP violations is closely tied to the monetary-fiscal policy mix. In particular, under active monetary policy, the model generates the classic short-horizon UIP puzzle regardless of tax policy, since debt always increases persistently following a contractionary monetary shock. Furthermore, if $\rho_\tau > \underline{\rho}(\kappa_b)$, then the equilibrium convenience yield inherits the cyclical dynamics of government debt, and the UIP violations reverse course at longer horizons, in line with the empirical evidence. Lastly, under passive monetary policy there are no UIP violations at any horizon, because inflation stabilizes debt, and thus the convenience yield differential as well. These results are formalized in Proposition 1 below.

PROPOSITION 1 (UIP Violations). *The magnitude and direction of the UIP regression coefficients* $\beta_k = \frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*}$ *depend on the monetary-fiscal policy mix as follows.*

(i) ***Active Monetary, Passive Fiscal policy*** $(\phi_\pi > 1, \kappa_b \in (\theta - \theta_2, \frac{\theta + (\theta_2 - 1)\rho_\tau}{1 - \rho_\tau}))$***:***

(a) $\rho_\tau \leq \underline{\rho}(\kappa_b)$*: UIP violations conform with the classic UIP puzzle at all horizons and decline monotonically to zero:*

$$\beta_k < 0 \text{ for } k = 1, 2, 3, \ldots$$

(b) $\rho_\tau > \underline{\rho}(\kappa_b)$*: UIP violations exhibit cyclical (cosine) dynamics, initially negative at short horizons, but eventually turning positive, i.e. there exists a* $\bar{k} > 1$ *such that*

$$\beta_k < 0 \text{ for } k < \bar{k}$$
$$\beta_k > 0 \text{ for some } k > \bar{k}$$

(ii) ***Passive Monetary, Active Fiscal policy*** $(\phi_\pi < 1, \kappa_b \in (0, \theta - \theta_2))$***:*** *UIP violations go in the same direction at all horizons and are in fact always zero:*

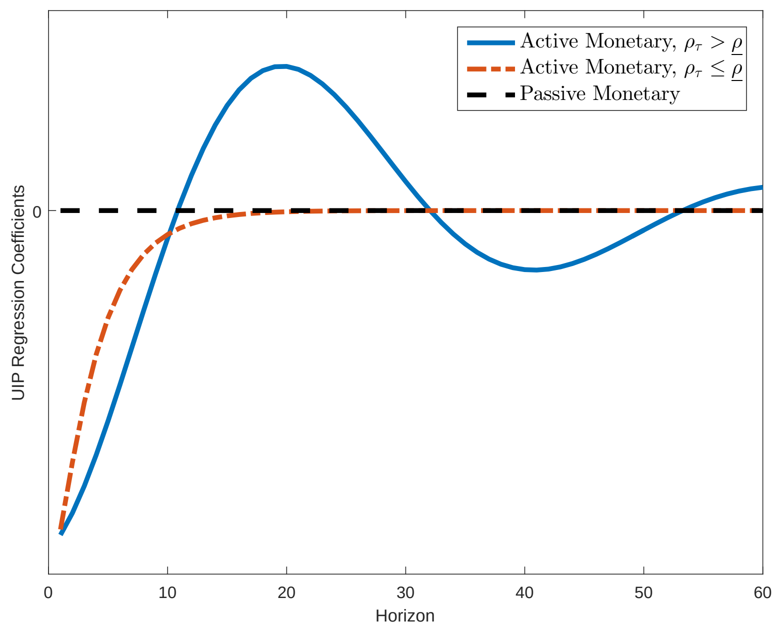
$$\beta_k = 0 \text{ for } k = 1, 2, 3, \ldots$$

Proof. See Online Appendix C.4. □

Proposition 1 derives the main analytical results, also illustrated in Figure 6, and showcases how the monetary-fiscal interaction determines the profile of UIP violations at different horizons. To better understand the intuition behind the results, it is useful to work through the response to a contractionary monetary shock. Under active monetary policy, the shock increases the interest rate and decreases inflation on impact. The fall in inflation leads to an increase in the outstanding amount of real government debt, which lowers its equilibrium convenience yield and leads to a compensating increase in the excess financial return on the home currency. Thus, the high interest rate coincides with high expected excess currency returns next period, which generates the classic UIP puzzle. Moreover, the increase in debt is persistent, and hence the negative UIP violations persist at short-horizons.

Whether the UIP violations reverse direction at longer horizons or not depends on the interaction of monetary and fiscal policy, but importantly the UIP reversals can occur only

Figure 6: Model Implied UIP Regression Coefficients



under an active monetary policy regime, which is consistent with the empirical evidence presented earlier. When monetary policy is active and taxes are relatively responsive, i.e. $\rho_\tau \leq \underline{\rho}(\kappa_b)$, then debt falls back to steady state in a monotonic fashion. The convenience yield differential follows a similar pattern, and thus the UIP violations themselves are also monotonic and we have $\beta_k < 0$ for all k. On the other hand, when tax policy is sluggish government debt has cyclical dynamics, and thus it falls below steady state before converging. As it does so, it becomes relatively scarce, which increases its marginal non-pecuniary value and pushes the home convenience yield above its steady state. In turn, this makes the foreign bond the relatively less desirable asset, and as a result the compensating equilibrium excess returns switch to the foreign currency. This generates a reversal in the UIP violations at longer horizons, and β_k turn positive.

On the other hand, if monetary policy is passive, a contractionary monetary shock leads to *higher* rather than lower inflation, which reverses the direction of the valuation channel and helps pay for the increased financing costs of the government. This stabilizes debt, and consequently also stabilizes the equilibrium convenience yield differential and excess currency returns. Hence, with passive monetary policy there are no UIP violations at any horizon.

Thus, the dynamics of UIP violations are tied to the interaction of monetary and fiscal policies, and the resulting speed and responsiveness of the government debt repayment mechanism. When debts are paid off through the most flexible mechanism, the inflation tax (passive monetary policy), debt is insulated from shocks, and hence the convenience yield is constant and there is no scope for UIP violations. On the other hand, when monetary policy is active, but taxes are relatively responsive ($\rho_\tau \leq \underline{\rho}(\kappa_b)$), debt has a controlled, monotonic response to shocks and the model generates the classic UIP puzzle relationship at all horizons. Lastly, when debt repayment depends on sluggish tax policy (active monetary policy and $\rho_\tau > \underline{\rho}(\kappa_b)$), the model generates cyclical debt dynamics, and correspondingly cyclical UIP violations that reverse course at longer horizons.

To sum up, the model can generate both the cyclical profile of UIP violations observed in the data, and the relationship with the monetary policy stance. Importantly, the result is not specific to a particular shock, but is due to the monetary-fiscal policy interaction, which determines the fundamental dynamics of the model.[17] Introducing more shocks and complications does not change the general message, as exemplified by the next section.[18]

5 Quantitative Model

Next, I relax the simplifying assumptions of the previous section and examine the quantitative performance of the mechanism, by setting it in a benchmark, nominal two country general equilibrium model in the spirit of Clarida et al. (2002). There are two symmetric countries, home and foreign. Households have access to a complete set of Arrow-Debreu securities and consume both a domestically produced final good and a foreign final good. Final goods sectors are competitive and aggregate domestically produced intermediate goods. The intermediate good firms are monopolistically competitive and face Calvo-type frictions in setting nominal prices. The government implements monetary policy by setting the interest rate and finances spending via lump-sum taxation and issuing government bonds.

5.1 Households

As in Clarida et al. (2002), the representative household maximizes the following utility,

$$E_t \sum_{j=0}^{\infty} \beta^j \left(\frac{C_{t+j}^{1-\sigma}}{1-\sigma} - \frac{N_{it}^{1+\nu}}{1+\nu}\right)$$

with consumption (C_t) a CES aggregate of home (H) and foreign (F) final goods,

[17] The crucial role played by cyclical debt dynamics is also borne out by the data. The empirical IRF of US debt follows a cyclical pattern similar to the one implied by the model – see Appendix D.5 for details.

[18] While the model abstracts from it, introducing trade in forward contracts does not change the results. The intuition is that forwards create a synthetic position that is long foreign bonds and short home bonds, and hence earns the respective convenience yield differential. Please see Appendix D.1 for details.

$$C_t = \left(a_h^{\frac{1}{\eta}} C_{Ht}^{\frac{\eta-1}{\eta}} + a_f^{\frac{1}{\eta}} C_{Ft}^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}$$

where η is the elasticity of substitution between the two goods and the weights a_h and a_f, normalized to sum to 1, determine the degree of home bias in consumption. C_{Ht} and C_{Ft} are the amount of the home final good and the foreign final good that the household purchases.

To motivate the demand for liquidity, I assume that the household incurs transaction costs in purchasing consumption, the standard approach in the quantitative literature on bond convenience yields (Bansal and Coleman (1996), Bansal et al. (2011)).[19] I model the transaction costs with a flexible CES function that includes both real money balances and real bond holdings as convenience assets:

$$\Psi(c_t, m_t, b_{ht}, b_{ft}) = \bar{\psi} c_t^{\alpha_1} h(m_t, b_{ht}, b_{ft})^{1-\alpha_1}$$

The transaction cost function has two components, the level of transactions C_t and a bundle of transaction services $h(m_t, b_{ht}, b_{ft})$, which is generated by the three convenience assets: real money balances m_t and real holdings of home and foreign nominal bonds b_{ht} and b_{ft}. Transaction costs are increasing in the level of purchases (C_t) and decreasing in the level of transaction services (i.e. $\alpha_1 > 1$). The transaction services $h(\cdot)$ are a CES aggregator of real money balances and a bundle of transaction services generated by bonds:

$$h(m_t, b_{ht}, b_{ft}) = (m_t^{\frac{\eta_m-1}{\eta_m}} + h^b(b_{ht}, b_{ft})^{\frac{\eta_m-1}{\eta_m}})^{\frac{\eta_m}{\eta_m-1}}$$

where

$$h^b(b_{ht}, b_{ft}) = \gamma(a_b b_{ht}^{\frac{\eta_b-1}{\eta_b}} + (1-a_b) b_{ft}^{\frac{\eta_b-1}{\eta_b}})^{\frac{\eta_b}{\eta_b-1}}$$

The nested structure of transaction services captures the idea that money and bonds are two separate classes of convenience assets and allows for different elasticity of substitution between money and the bundle of bonds (η_m), and between home and foreign bonds (η_b). The parameter γ controls the relative importance of bonds versus money as convenience assets, and the parameter a_b controls the relative importance of home to foreign bonds.

[19] Here I opt for transaction costs, rather than "bonds-in-the-utility", in order to be directly comparable to the previous quantitative literature. In any case, the two approaches are equivalent (see Feenstra (1986)). Moreover, the results do not change if one uses a cash-in-advance framework instead.

The budget constraint of the household is

$$C_t + \int \Omega_{H,t}(z_{t+1}) x_t(z_{t+1}) dz_{t+1} + \Psi(c_t, m_t, b_{ht}, b_{ft}) + m_t + b_{ht} + b_{ft} =$$
$$w_t N_{it} + \frac{x_{t-1}(z_t)}{\Pi_t} - \tau_t + d_t + \frac{m_{t-1}}{\Pi_t} + b_{h,t-1}\frac{(1+i_{t-1})}{\Pi_t} + b_{f,t-1}\frac{(1+i^*_{t-1})}{\Pi_t}\frac{S_t}{S_{t-1}}$$

where $\Omega_{H,t}(z_{t+1})$ is the home currency price of the Arrow-Debreu security that pays off in the state of nature z_{t+1} and $x_t(z_{t+1})$ is the amount of this security that the home household has purchased. The household spends money on consumption, Arrow-Debreu securities, transaction costs, money holdings, and home and foreign nominal bonds. It funds purchases with money balances it carries over from the previous period, real wages w_t, profits from the intermediate good firms d_t, and payoffs from its holdings of contingent claims, and home and foreign bonds. Lastly, it also pays lump-sum taxes τ_t to the domestic government.

This first-order necessary conditions for home and foreign nominal bond holdings are:

$$1 = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1 + \Psi_c(c_t, m_t, b_{ht,} b_{ft})}{1 + \Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1,} b_{f,t+1})} \frac{1}{\Pi_{t+1}} \frac{1 + i_t}{1 + \Psi_{b_h}(c_t, m_t, b_{ht}, b_{ft})} \tag{13}$$

$$1 = \beta E_t \left(\frac{C_{t+1}}{C_t}\right)^{-\sigma} \frac{1 + \Psi_c(c_t, m_t, b_{ht,} b_{ft})}{1 + \Psi_c(c_{t+1}, m_{t+1}, b_{h,t+1,} b_{f,t+1})} \frac{S_{t+1}}{\Pi_{t+1} S_t} \frac{1 + i^*_t}{1 + \Psi_{b_f}(c_t, m_t, b_{ht}, b_{ft})} \tag{14}$$

where the term $\Psi_x = \frac{\partial \Psi}{\partial x}$ is the derivative of the transaction costs in respect to the variable x. The terms Ψ_{b_h} and Ψ_{b_f} are the marginal transaction benefit of holding an extra unit of home and foreign bonds respectively. Similarly to the analytical model, these marginal benefits determine the convenience yields and will generate deviations from UIP.

5.2 Firms

There is a home representative final goods firm which uses the domestic continuum of intermediate goods and the following CES technology to produce total output $Y_{H,t}$:

$$Y_{H,t} = \left(\int_0^1 Y_{it}^{\frac{\xi-1}{\xi}} di\right)^{\frac{\xi}{\xi-1}}.$$

Profit maximization yields the standard CES demand and price index

$$Y_{it} = \left(\frac{P_{it}}{P_{Ht}}\right)^{-\xi} Y_{Ht} \ ; \ P_{H,t} = \left(\int_0^1 P_{it}^{1-\xi} di\right)^{\frac{1}{1-\xi}}$$

Intermediate goods firms use a production technology linear in labor, $Y_{it} = A_t N^D_{it}$, where A_t is an exogenous productivity process that is AR(1) in logs. The firms face a Calvo friction and have a probability $1-\theta$ of being able to adjust prices. Firms that adjust choose their optimal price $\bar{P}_t$, and firms that do not get to re-optimize keep their prices constant. Hence, the price of the home final good evolves according to

$$P_{Ht} = (\theta P_{H,t-1}^{1-\xi} + (1-\theta)\bar{P}_t^{1-\xi})^{\frac{1}{1-\xi}} \tag{15}$$

5.3 Government

The government consists of a Monetary Authority (MA), and a separate Fiscal Authority (FA).[20] The MA follows a standard Taylor rule (in log-approximation to steady state):

$$\hat{i}_t = \rho_i \hat{i}_{t-1} + (1-\rho_i)\phi_\pi \hat{\pi}_t + v_t$$

where π_t is CPI inflation and v_t is an iid monetary shock. The MA issues the supply of the domestic currency, M_t^s, and backs it with holdings of domestic government bonds, so that $M_t^s = B_{ht}^M$ where B_{ht}^M is the amount of domestic bonds held by the Central Bank. The MA transfers all seignorage revenues to the FA and faces the budget constraint

$$T_t^M = M_t^s - M_{t-1}^s + B_{h,t-1}^M(1+i_{t-1}) - B_{ht}^M,$$

where T_t^M is the money transferred to the Fiscal Authority.

The Fiscal Authority collects taxes, the seignorage from the MA, and issues government bonds to fund a constant level of real expenditures (g) and faces the budget constraint

$$B_{ht}^G + T_t + T_t^M = B_{h,t-1}^G(1+i_{t-1}) + P_t g$$

where B_{ht}^G is nominal government debt and T_t are nominal lump-sum taxes. Lastly, I follow the quantitative literature on the interaction between monetary and fiscal policy, and model tax policy (as percent of GDP) as a simple rule linear in debt-to-GDP:[21]

$$\frac{P_t \tau_t}{P_{H,t} Y_{H,t}} = \rho_\tau \frac{P_{t-1}\tau_{t-1}}{P_{H,t-1}Y_{H,t-1}} + (1-\rho_\tau)\kappa_b \frac{P_{t-1} b_{h,t-1}^G}{P_{H,t-1}Y_{H,t-1}}$$

[20] Using a single consolidated government setup instead has no effect on the results.

[21] See for example Leeper (1991), Davig and Leeper (2007), and Bianchi and Ilut (2013) among others.

5.4 Excess Currency Returns and UIP violations

Log-linearize (13) and (14) and combine them to obtain

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \left|\frac{\Psi^H}{1+\Psi^H}\right| \hat{\Psi}_t^H - \left|\frac{\Psi^F}{1+\Psi^F}\right| \hat{\Psi}_t^F \quad (16)$$

where hatted variables represent log-deviations from steady state. As before, the term $\left|\frac{\Psi^H}{1+\Psi^H}\right| \hat{\Psi}_t^H$ denotes the home convenience yield, and thus expected excess currency returns equal the convenience yield differential. Similarly, the convenience yield has a fundamentally negative relationship with the level of the interest rate, as discussed previously.

Moreover, since at the symmetric steady state $\Psi^H = \Psi^F$, (16) reduces further to

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}_t^* - \hat{i}_t) = \frac{1}{\eta_b} \left|\frac{\Psi^H}{1+\Psi^H}\right| (\hat{b}_{ft} - \hat{b}_{ht}) \quad (17)$$

Hence, the equilibrium convenience yield differential is again linked to bond holdings, but this time to the relative holdings of foreign and home bonds. The more abundant are home bonds, relative to foreign bonds, the lower is the relative marginal convenience value of holding an extra unit of the home bonds, and thus the lower is the convenience yield differential.[22]

5.5 Main Quantitative Results

The model is log-linearized around the symmetric, zero-inflation steady state, and calibrated to standard parameters targeting unconditional, non-UIP related moments.

5.5.1 Calibration

The benchmark calibration is presented in Table 1, with one period in the model representing one quarter. I set risk aversion σ equal to 3, $\beta = 0.9901$ (implying an annual steady state real interest rate of 3%), and the inverse Frisch elasticity of labor supply $\nu = 1.5$, all of which are standard values in the RBC literature. Estimates of the elasticity of substitution between home and foreign goods vary, but most fall in the range from 1 to 2 and I follow Chari et al. (2002) and set $\eta = 1.5$. I set the elasticity of substitution between domestic goods, ξ, equal to 7.66, implying markups of 15%, and choose the degree of home bias $a_h = 0.8$, a common value in the literature that is roughly in the middle of the range of values for the G7 countries.

[22] Note that the *level* of the convenience yield also depends on demand factors (i.e. consumption), and on the supply of the other liquid asset, money. However, what matters for excess currency returns is the convenience yield *differential*, and in this case these other effects cancel out because they affect the liquidity values of both home and foreign bonds.

In calibrating the transaction cost function, I set $\alpha_1, \eta_m, \gamma, \bar{\psi}$ to match the interest rate semi-elasticity of money demand, the income elasticity of money demand, money velocity and the average convenience yield. I target an interest rate semi-elasticity of money demand of 7, roughly in the middle of most estimates, which range from 3 to 11 (see discussion in Burnside et al. (2011)). I set the income elasticity of money demand to 1, and the money velocity equal to 7.7, which is the average value for the $M1$ money aggregate in the US for the time period $1976 - 2013$. Next, I target a steady state annualized convenience yield of 1%, which is in the middle of the range of estimates in the literature.[23] Finally, I choose a_b so that foreign bonds constitute 10% of the steady state bond portfolios of the households, implying a strong home bias in accordance with the data.[24]

Table 1: Calibration

Param	Description	Value	Param	Description	Value
σ	Risk Aversion	3	$\frac{G}{Y}$	Gov Expenditures to GDP	0.22
ν	Inverse Frisch Elast	1.5	$\frac{b_h^D}{Y}$	Gov Debt to GDP	0.5
η	Elast Subst Consumption	1.5	θ	Calvo Parameter	0.667
a_h	Home Bias in Consumption	0.8	ϕ_π	Taylor Rule Inflation Response	1.5
β	Time Discount	0.9901	ρ_i	Taylor Rule Smoothing	0.9
ξ	Elast Subst Dom Goods	7.66	σ_v	Std Dev Monetary Shock	0.0033
α_1		19			
η_m	Elast Subst b/w Bonds and m_t	0.1	ρ_τ	Tax Smoothing	0.92
γ		0.425	κ_b	Tax Response to Debt	0.48
$\bar{\psi}$		4.2e-18			
a_b	Home Bias in Bond Holdings	0.9998	ρ_a	Autocorrelation TFP	0.97
η_b	Elast Subst b/w H and F Bonds	0.25	σ_a	Std Dev TFP shock	0.0078

There is little prior literature guidance in choosing η_b, the elasticity of substitution between home and foreign bonds, so I set it equal to 0.25 to match the US data on the volatility of foreign bond holdings to GDP. In the model, increasing η_b makes the home and foreign bonds better substitutes and increases the overall volatility of foreign bond holdings.

I calibrate the steady state ratio of government spending to GDP to 22% and the ratio of government debt to GDP to 50%, the average values of total federal spending to GDP and total federal debt to GDP, respectively, in US data. For the Taylor rule I set $\phi_\pi = 1.5$, and pick $\rho_i = 0.9$ to match the persistence of the US interest rate.[25] Lastly, I estimate the postulated tax rule using US data on federal taxes and debt, and obtain $\rho_\tau = 0.92$ and

[23] Krishnamurthy and Vissing-Jorgensen (2012) estimate that the average convenience yield on Treasuries is between 85 and 166 bp, while Krishnamurty (2002) finds an average Treasury convenience yield of 144 bp.

[24] See Warnock and Burger (2003), Fidora et al. (2007), Coeurdacier and Rey (2013)

[25] Bianchi and Ilut (2013) also estimate a value of 0.9 for the Taylor Rule smoothing parameter.

$\kappa_b = 0.48$. The Calvo parameter is set to $\theta = 0.667$.

For the TFP process, I estimate a AR(1) in logs using John Fernald's TFP data and get $\rho_a = 0.97$ and $\sigma_a = 0.0078$. I back out the standard deviation of the Taylor rule shock from the US data as well, using data on the federal funds rate, CPI inflation and the calibrated parameters of the Taylor rule to construct a series of residuals. The standard deviation of the residuals leads me to $\sigma_v = 0.0033$.[26]

5.5.2 UIP Violations

I examine the model's quantitative ability to match the data in two ways. First, I compute the model implied UIP coefficients from the UIP regressions,

$$\hat{\lambda}_{t+k} = \alpha_k + \beta_k(\hat{i}_t - \hat{i}_t^*) + \varepsilon_{t+k},$$

where $\hat{\lambda}_{t+k} = \hat{s}_{t+k} - \hat{s}_{t+k-1} + \hat{i}^*_{t+k-1} - \hat{i}_{t+k-1}$, and compare the coefficients β_k with their empirical counterparts. Second, I examine the underlying exchange rate behavior by estimating

$$\hat{s}_{t+k} - \hat{s}_t = \alpha_k + \gamma_k(\hat{i}_t - \hat{i}_t^*) + \varepsilon_{t+k},$$

the same direct projections as in the empirical section. Recall that the sequence $\{\gamma_k\}$ provides an estimate of the IRF of the exchange rate to an innovation in the interest rate differential.

The left panel of Figure 7 shows the results from the UIP regressions. The solid blue line plots the β_k coefficients implied by the model, and the dashed line plots the empirical estimates.[27] The model matches the data quite well – it generates negative UIP violations at horizons of up to 3 years, and positive violations at horizons between 4 and 8 years. The model is especially successful at generating the longer-horizon positive UIP violations and the overall cyclical profile of the violations. It matches the timing of the switch in signs, and on average it can account for more than three-quarters of the magnitude of the β_k estimates at horizons longer than 1 year. Thus, we see that the model is very good at generating the non-linear, low-frequency variation in the UIP violations that underpin the reversal of the puzzle. Importantly, these plots summarize not a single moment, but a whole collection of sixty different moments, and none of them were targeted in the calibration.

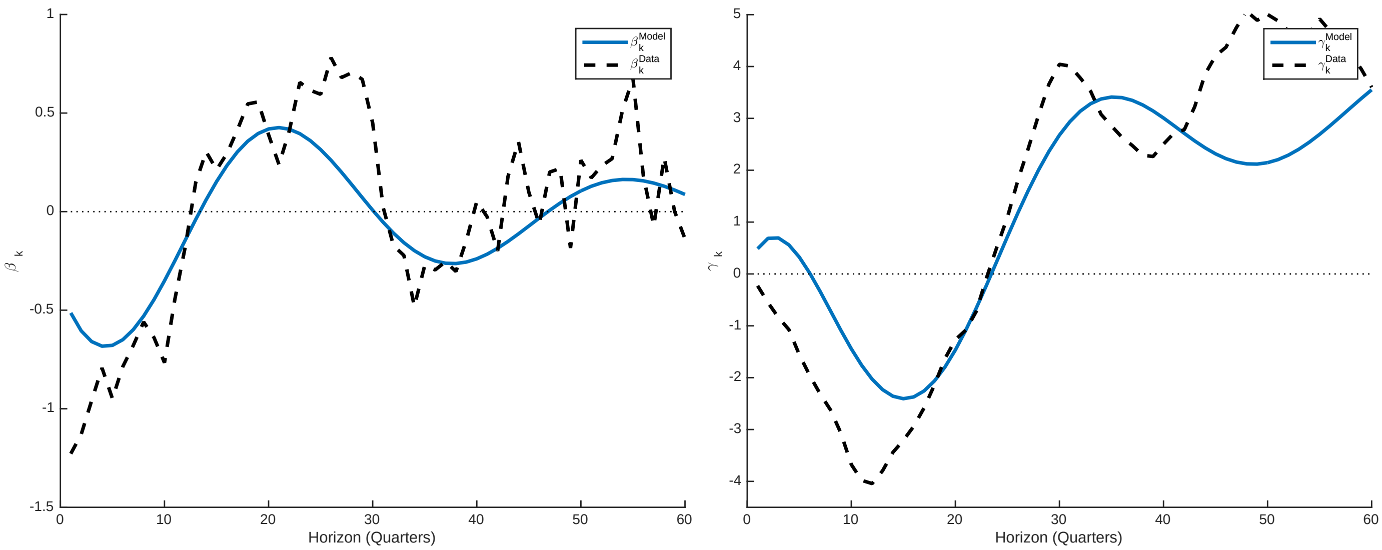
The right panel of Figure 7 plots the γ_k estimates, and shows that the model-implied exchange rate dynamics also align closely with the data, with an initial appreciation followed

[26] This implies that a one std dev. monetary shock results in a 19bp response on impact by the interest rate, matching the estimate in Eichenbaum and Evans (1995). Moreover, $\sigma_v = 0.0033$ is among the range of common estimates, e.g. 0.0036 in Davig and Leeper (2007) and 0.0030 in Galí and Rabanal (2005).

[27] To be conservative, I use the empirical estimates for the subset of currencies with stronger monetary policy, since they exhibit the biggest violations, and the model itself is calibrated to an active MP regime.

Figure 7: Regression Estimates, Model vs Data

(a) UIP Regression Coefficients (b) Exchange Rate IRF



by a strong depreciation. Thus, the model delivers cyclical UIP violations, thanks to non-monotonic exchange rate behavior, just like in the data.

The general equilibrium model has more moving parts than the analytical model of the previous section, but the main mechanism underlying the UIP violations and the non-monotonic exchange rate dynamics is the same. Contractionary shocks, either monetary or TFP, lower inflation and GDP (and thus tax revenues), and this leads to an increase in the real stock of home debt. As home debt becomes less scarce, its marginal liquidity value falls and as a result the home currency earns compensating excess returns in equilibrium. This generates the classic UIP Puzzle that high interest rates today are associated with higher expected excess currency returns. In turn, the combination of active monetary policy and a sluggish tax policy delivers cyclical debt dynamics (for the same reasons as in the analytical model), and as a result the direction of the UIP violations reverses at longer horizons.

The main difference with the analytical model is that now both home and foreign debt markets are in equilibrium, and hence there is an adjustment in the holdings of foreign debt as well. In particular, as home investors increase their holdings of home debt, the marginal liquidity value of debt instruments as a whole declines, and this leads them to demand less foreign debt. In essence, home investors are substituting into home debt, and out of foreign debt, which tilts their overall portfolio composition towards home debt, and hence increases the marginal value of foreign debt *relative* to home debt (see eq. (17)). This serves as a reinforcing effect that further decreases the convenience yield differential.

To sum up, the model delivers cyclical UIP violations that match the data very well.

The cyclicality can be traced back to the fact that monetary policy does not accommodate fiscal policy, which leads to cyclical fluctuations in the relative supply of liquid assets.

5.5.3 Unconditional Moments

Table 2: Unconditional Moments

	Data	Benchmark Model	Monetary Shocks Only	TFP Shocks Only	No Convenience Yield
Standard Deviations					
Δs_t	5.60	2.96	2.95	0.25	3.23
$i_t - i_t^*$	0.65	0.32	0.26	0.17	0.21
Autocorrelations					
Δs_t	0.09	-0.02	-0.02	0.24	-0.04
$i_t - i_t^*$	0.74	0.8	0.73	0.98	0.70
Macro Aggregates :					
Standard Deviations					
Δy_t	0.78	1.06	0.99	0.39	1.18
Δc_t	0.62	0.45	0.44	0.23	0.53
$\Delta(b_t^g/y_t)$	3.15	2.49	2.42	0.58	2.18
i_t	0.84	0.44	0.28	0.34	0.30
Autocorrelations					
Δy_t	0.232	-0.18	-0.26	0.32	-0.19
Δc_t	0.43	-0.11	-0.24	0.25	-0.14
$\Delta(b_t^g/y_t)$	0.34	0.42	0.4	0.9	0.37
i_t	0.86	0.88	0.73	0.99	0.80

Notes: Standard deviations are expressed in percentage terms. The data on domestic variables is for the US, the data on international variables is for the US against the other countries in the sample. The second column presents the results of the benchmark calibration, columns three and four present results when only monetary and TFP shocks are active, and column five shuts down the convenience yield mechanism.

For the regression results in the previous section to be fully meaningful, it is important that the model also delivers appropriate unconditional moments for the key variables. To verify this, Table 2 presents the corresponding moments, with the second column reporting the data moments, and the third column the moments of the benchmark calibration of the model. The data on domestic variables is for the US, given that the calibration targeted US data, and the exchange rate moments are the average of all currencies against the USD. Except for the autocorrelation of i_t, the moments in the table were not directly targeted by the calibration, hence they can be viewed as over-identifying restrictions.

Most importantly, the model is successful in matching the relative volatility of exchange rate changes to interest rate differentials, which is 8.6 in the data and 9.2 in the model, and their respective autocorrelations. This is especially re-assuring for the regression results of the previous section, and also means that the model is not only able to match the conditional dynamics of these two variables, but also their unconditional moments.[28]

Second, the model also reproduces the dynamics of government debt, matching both the volatility and persistence of Debt-to-GDP despite not targeting either. In the data, the std. dev. of $\Delta(b_t^g/y_t)$ is 3.15% with an autocorrelation of 0.34, and the model delivers an implied std. dev. of 2.49% and autocorrelation of 0.42. Matching the dynamics of government debt is crucial as it plays a key role in driving the convenience yield in the model, and we want to ensure that the model does not produce accurate UIP violations due to unreasonable government debt behavior. The autocorrelation is particularly important, as it speaks to the cyclical dynamics of debt, which underpin the reversal of UIP violations in the model.

The model matches the unconditional volatility of the standard macro aggregates reasonably well, although it slightly overshoots the volatility of output and undershoots the volatility of consumption. It also has some trouble matching the persistence in the growth rates of output and consumption, both of which are positively autocorrelated in the data, but mildly negatively autocorrelated in the model. However, these are well-known issues with the standard two-country New-Keynesian model, and not specific to the introduction of the convenience yield mechanism. This can be seen from the last column of Table 2, which computes the model's moments when the liquidity value of bonds is shut-down (and thus UIP holds at all horizons). Comparing with the benchmark calibration, we see that adding the convenience yield in fact improves the fit – thus the model does not deliver success on the UIP front at the expense of other things.

Overall, the convenience yield mechanism fits seamlessly into the standard two-country framework, and helps deliver appropriate exchange rate and interest rate dynamics, without hurting the implications about macro aggregates. This result is likely related to the observation that the long-run behavior of the exchange rate is roughly consistent with UIP, because the negative and positive UIP violations cancel each other over time (as they do in the data). In turn, the long-run dynamics of the model, and hence the unconditional macro moments, are also quite close to the model where UIP holds at all horizons.

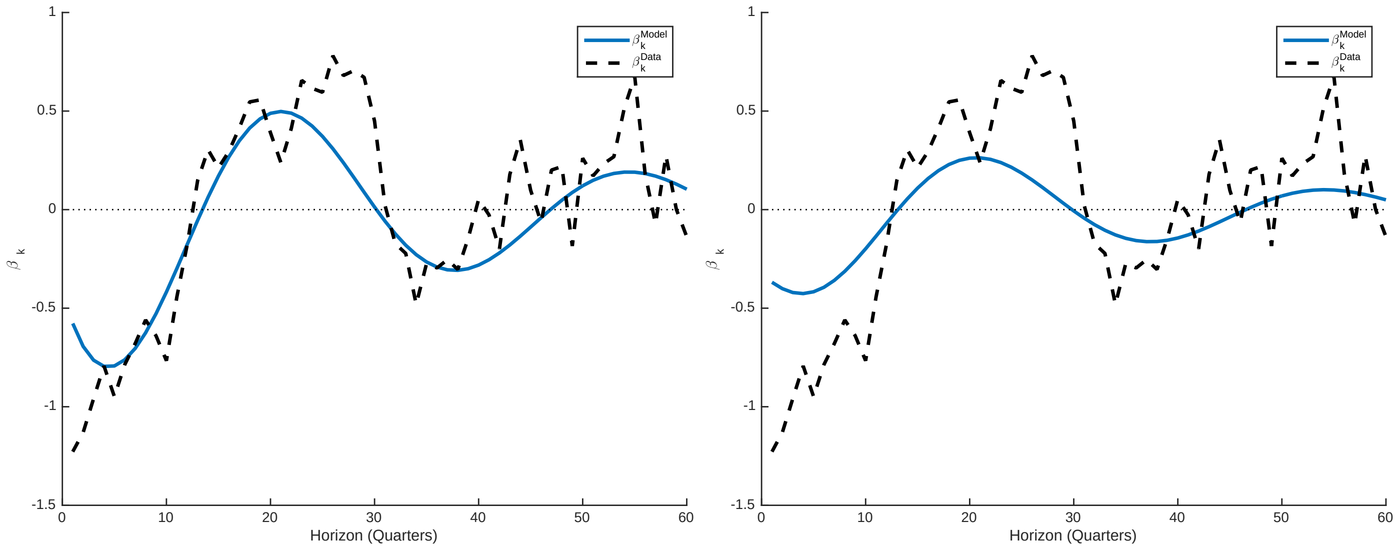
Lastly, note that the model does not have counter-factual implications about the behavior of interest rates on assets that do not posses a convenience yield. In the data, interest rate differentials of a variety of short-term debt instruments (Treasuries, LIBOR, etc.) behave similarly, even though these assets are likely to have different degrees of convenience

[28]Still, the model is only able to explain half of the absolute volatility of exchange rates and interest rate differentials. Perhaps, this is something that could be alleviated by considering more shocks, or introducing cointegrated TFP shocks, as in Rabanal et al. (2011)

Figure 8: Regression Estimates, Model vs Data

(a) Monetary Shocks Only

(b) TFP Shocks Only



benefits. This is also true in the model, and the reason is that differences in the convenience yield show up primarily in the level of interest rates, but not in their *dynamics*, which is what matters for the UIP violations. Moreover, the model can easily incorporate long-term bonds and would imply that the returns on long-term bonds are equalized across countries, while those on short-term bonds are not, *even if* we assume that long-term bonds earn the same convenience yield as short-term ones. For more details, please see Online Appendix D.

5.5.4 System Dynamics vs Shocks

The cyclical pattern of UIP violations in the model is not a function of a particular type of shock, but is a result of its non-linear equilibrium dynamics. To make this point clear, Figure 8 compares the model-implied β_k when TFP shocks are shut-down, in panel a), and when monetary shocks are shut-down in panel b). Regardless of the source of exogenous variation, the model's implications are very similar and quite close to the data.

This is due to the fact that the convenience yield mechanism introduces two fundamental features to the equilibrium of the model. First, the convenience yield has a negative relationship with domestic interest rates, and a positive one with excess currency returns. Again, this is because as the liquidity value of home debt falls, there is a rise in the domestic interest rate that compensates for the fact that the bond's non-pecuniary value relative to money has fallen, and also a rise in the excess return on the home currency that compensates for the fact that the non-pecuniary value of the bond has fallen *relative* to the foreign one.

Second, the subsequent dynamics of the excess currency returns are determined by the dynamics of government debt. Due to the combination of active monetary policy and sluggish taxes, debt has cyclical dynamics, and falls below steady state before converging. As home debt falls below steady state, it becomes relatively scarce and the compensating excess currency return, switches to the foreign currency, generating a reversal in the direction of UIP violations. Thus, the basic mechanism behind the UIP violations, and their overall profile at different horizons, is not tied to any particular source of exogenous variation. Moreover, the importance of the cyclical debt dynamics is borne out by the data as well. I estimate the empirical dynamics of US debt in Appendix D.5, and find that it follows an oscillating, cyclical type of an impulse response similar to the one implied by the model.

Still, including both types of shocks is important for producing realistic unconditional moments, as shown by Table 2. For example, monetary shocks produce excessively volatile exchange rates (relative to interest rate differentials), while TFP shocks have the opposite problem, of producing too little exchange rate volatility. Thus, even though both shocks can generate the correct pattern of UIP violations on their own, it is the combination of the two that delivers appropriate unconditional moments.

5.6 Model Discussion

I conclude the analysis of the model with a short-discussion. First, note that the key to generating the UIP coefficients β_k is the time-variation in the equilibrium convenience yield. Since this is driven by variation in the stock of government debt, what matters are the log-deviations of debt from its steady state, and not its overall level. The model can generate significant β_k estimates both for countries that have high overall level of debt (i.e. US), and countries with lower stocks of debt – what matters are movements in the percentage deviation from steady state (or trend in the data).[29]

Relatedly, the assumption that all debt is short-term debt is innocuous, and introducing long-term bonds will in fact only strengthen the results. This is again because the model is driven by log-deviations of the relevant debt variable from its trend, not from movements in absolute dollar figures. Hence, even though only a small fraction of total government debt is in terms of very liquid, short-term bonds, and thus that component has a relatively small standard deviation in terms of *absolute* dollar amounts, it is in fact the most volatile component of debt in terms of *log-deviations* from trend. Introducing slow-moving long-term debt in the model would make the short-term debt more volatile in terms of log-deviations from steady state, and will only strengthen the main mechanism. Moreover, decomposing the empirical UIP violations into a pure exchange rate and a term-structure component shows

[29] Differences in the average level of debt across countries will show up in unconditional premia, and not in the conditional premia captured by the UIP regressions. This is an interesting topic for future research.

that any term-structure effects are of secondary importance – see Appendix D.4 for details.

It is also important to emphasize that in the model a monetary shock affects debt due to both a valuation effect coming from inflation and an interest rate effect. Higher interest rates increase the financing cost of the government and add to the overall debt burden, while lower inflation increases the *real* value of outstanding debt. Quantitatively, the valuation channel is the most important one in the model, and accounts for the majority of debt and convenience yield fluctuations. Thus, even if the interest rate channel is not very strong in the data (since financing costs tend to be a small portion of government budgets), this is not an issue for the model, because the results are mainly driven by the valuation channel.

The model is also related to a couple of recent works, Engel (2016) and Itskhoki and Mukhin (2016), which analyze UIP violations due to exogenous shocks to liquidity. Engel (2016) looks specifically at the changing sign of UIP violations and argues that a model driven by the combination of volatile, but transitory shocks to the value of liquidity and persistent TFP shocks (that act as shocks to the real exchange rate) can explain this new puzzle. In his framework, the shocks to liquidity dominate the covariance structure at short-horizons and generate the classic UIP puzzle, while the persistent TFP shocks drive the positive violations at longer horizons. The key differences between that paper and the model presented here are two-fold. First, the economic mechanisms at play are quite different. Most importantly, in Engel (2016) the supply of bonds is exogenous and only home bonds provide liquidity services, while I endogenize the supply of bonds by modeling fiscal policy and allow both home and foreign bonds to provide liquidity services. Second, I show that this enriched economic mechanism can generate all salient results through endogenous fluctuations in the equilibrium convenience yields, and the change in the sign of the UIP violations is due to non-linear dynamics and not due to a specific shock, or combination of shocks. On the other hand, Itskhoki and Mukhin (2016) find that small exogenous shocks to asset demand, which are isomorphic to exogenous shocks to the convenience yield itself, can help solve a number of puzzles with the standard IRBC model.

Lastly, the model abstracts from risk considerations, and analyzes only first-order effects, but it can easily be augmented with time-varying risk-premium mechanisms (e.g. Verdelhan (2010), Bansal and Shaliastovich (2012), Colacito and Croce (2013)). Since these mechanisms operate through higher-order terms, they will reinforce the results presented here, and lead to even stronger quantitative effects. In fact, as we saw the convenience yield mechanism could use some amplification in terms of short-horizon negative UIP violations. Thus, combining this model, which is very good and generating the low-frequency non-linear dynamics in UIP violations, with a source of high-frequency risk-premium variation could be particularly effective. This is an interesting direction for future research.

6 Government Debt and UIP Violations in the Data

Next, I turn to a direct test of the model, by extending the methodology of Krishnamurthy and Vissing-Jorgensen (2012) to the international setting. That paper proxies for the convenience yield by regressing the stock of government debt on the interest rate spread between Treasuries and other, less liquid and safe assets while also controlling for confounding effects.

A similar strategy can also be applied here. By equation (17), the equilibrium excess currency return in the model is a function of the relative holdings of home and foreign bonds:

$$E_t(\hat{s}_{t+1} - \hat{s}_t + \hat{i}^*_t - \hat{i}_t) = \left|\frac{\Psi^H}{1+\Psi^H}\right| \hat{\Psi}^H_t - \left|\frac{\Psi^F}{1+\Psi^F}\right| \hat{\Psi}^F_t = \frac{1}{\eta_b}\left|\frac{\Psi^H}{1+\Psi^H}\right| (\hat{b}_{ft} - \hat{b}_{ht}).$$

And while data on private portfolios is not readily available, I follow prior research and use the supply of home and foreign government debt as proxies.[30] In addition to government debt, I also consider the stock of commercial paper as a regressor, to control for possible substitution effects between high quality public and private debt.[31] Thus, I estimate

$$\lambda_{j,t+1} = \alpha_j + \beta(i_t - i^*_{j,t}) + \gamma \ln(\text{Debt}_t) + \gamma^* \ln(\text{Debt}^*_t) + \delta \ln(\text{CP}_t) + \text{Additional Controls} + \varepsilon_{j,t+1}$$

as a panel regression with fixed effects. As additional controls, I use the same set of variables as in Krishnamurthy and Vissing-Jorgensen (2012) – the slope of the yield curve and the stock market volatility, both for the US and for the relevant foreign country in each bilateral relation. Following the equilibrium condition of the model, eq. (17), I include the debt variables in real terms, after removing a deterministic exponential time-trend. However, as a robustness check, I also re-estimate all specifications using debt-to-GDP ratios instead, and all results remain the same – please see Appendix E for details.

Due to availability of data on quarterly foreign debt, the sample for this analysis starts in 1991. With the exception of the Deutsche Mark, this leaves the Euro legacy countries with a short sample size of at most 8 years of data (differing slightly due to government debt availability), and hence I drop them from the benchmark specification. Thus, the data for the benchmark results spans 1991-2013 for the 10 non-Euro currencies, including the German Deutsche Mark.[32] However Appendix E shows that the results are robust to

[30] In the model itself, private portfolio holdings track overall supply of debt closely.

[31] Commercial Paper is very short-term (< 1 year) unsecured debt of large firms with excellent credit ratings. With virtually zero default rate, it is a very safe investment that could also offers significant convenience benefits (e.g. Bansal et al. (2011)).

[32] To maximize the data and keep as close as possible to the original empirical analysis in Section 2.1, I consider 1-month excess currency returns at the daily frequency. I use quarterly debt to create daily frequency debt series, by using last quarter's debt to fill-in the daily values for the current quarter. Thus, the debt observation for March 31 is used for all days in April, May and June. This avoids look ahead bias, and ensures that the regressors contain at most time t information. As a robustness check, I re-estimate all

extending the sample - there I re-estimate all regressions omitting foreign debt, which is the main constraining factor, and I am thus able to extend the sample to 1984.

Table 3 reports the estimation results. In the left panel, I report estimation results on the whole sample, which includes both the financial crisis and the post-crisis zero interest rate environment. There is good reason to believe that this latter part of the sample is a period in which the convenience yield mechanism is not very strong. In the current zero interest rate environment, liquidity needs are fairly well satiated and the convenience yield is near-zero, while during the peak of the crisis period excess returns were likely predominantly driven by risk-premium related events. To explore this potential difference, in the right panel I report estimation results excluding the post-2007 period.

Table 3: Excess Currency Returns and Debt

	1991 - 2013				1991 - 2007			
	(1)	(2)	(3)	(4)	(1′)	(2′)	(3′)	(4′)
$i_t - i_t^*$	-1.4***	-1.53***	-0.60	-1.86*	-1.83***	-1.97***	-0.65	-0.08
	(0.46)	(0.47)	(0.49)	(0.96)	(0.49)	(0.49)	(0.51)	(0.52)
ln(Debt)		-1.42**	-5.56***	-4.88***		-2.65***	-7.85***	-7.71***
		(0.69)	(1.54)	(1.56)		(0.70)	(1.57)	(1.91)
ln(Debt*)		0.13	0.35**	0.19		0.18	0.33*	0.39**
		(0.18)	(0.17)	(0.14)		(0.17)	(0.17)	(0.15)
ln(CP)			-2.18***	-1.71**			-3.24***	-3.32**
			(0.68)	(0.74)			(0.83)	(1.32)
Add. Controls	No	No	No	Yes	No	No	No	Yes
# Currencies	10	10	10	10	10	10	10	10
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Estimates with Driscoll and Kraay (1998) standard errors robust to heteroskedasticity, serial correlation and cross-equation correlation. The debt stock variables are exponentially detrended. ***, ** and * denote significance at the 1%, 5% and 10% level respectively.

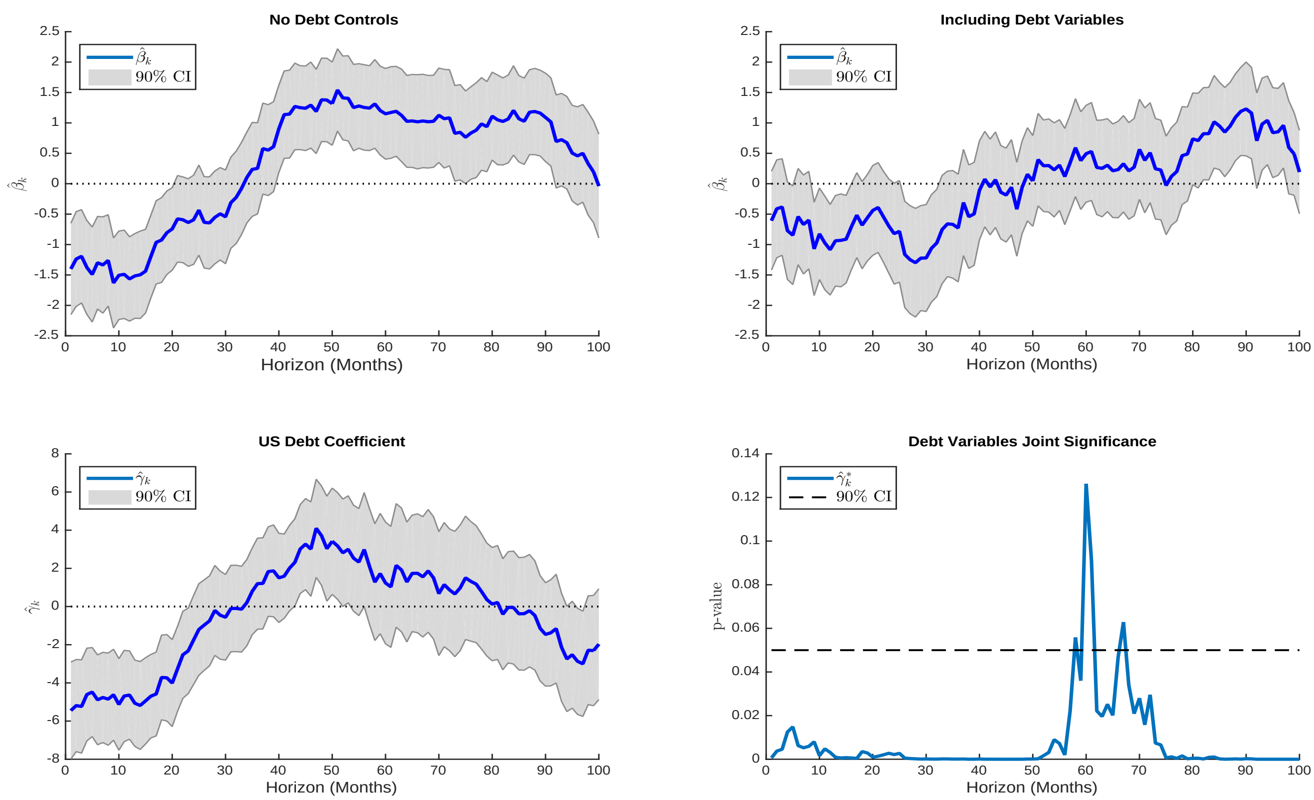
The results in both panels strongly support the model, but indeed the support is especially strong in the pre-crisis period. In all specifications, the coefficient on US debt is negative and significant, which signifies that just like in the model, in the data times of high US government debt are associated with high excess returns on the USD. The estimates are also economically significant, as they imply that a one standard deviation increase in US debt is associated with a 60bp increase in the excess return on the USD. This is comparable with the relationship between excess returns and the interest rate differential (as seen in Column

specifications at the quarterly frequency and the results remain the same – for details see Appendix E.

(1)), which implies that a one standard deviation increase in the interest rate differential is associated with a 40bp increase in the excess return on the USD.

Similarly consistent with the model, in all specifications the coefficient on foreign debt is positive, and is significant in 3 of the 6 specifications. However, it is interesting to note that it is an order of magnitude lower than the coefficient on US debt. This suggests that in the data the mechanism operates primarily through the effects of US debt on the US convenience yield, which is intuitively appealing given the special role that the US dollar plays in the international financial system.[33]

Figure 9: UIP Violations



Lastly, while there is a large variety of models that can rationalize the classic UIP puzzle, a unique differentiating feature of this model is that it can also deliver the reversal of UIP violations at longer horizons. With that in mind, next I augment the $k-$horizon UIP regression (eq. (2)) with the debt variables considered in this section and plot the resulting coefficients in Figure 9. Due to the shortened sample, I consider $k \leq 100$ months.

[33] Also, Hassan and Mano (2015) find that the standard, one-step ahead UIP regression coefficients are primarily driven by a common USD factor that drives all currencies against the dollar.

Several interesting results emerge. First, in the top left panel we see that the reversal of UIP coefficients is a pronounced feature of the shortened sample as well, with the magnitude and the timing of the reversal being virtually identical to the previous estimates. On the top right, however, we see that including the debt variables removes the reversal from the coefficient on the interest rate differential. Thus, controlling for the stock of debt reduces not only the short-horizon UIP estimates (as also evidenced by Table 3), but also eliminates the reversal at longer horizons. The bottom left panel shows that this effect on the interest rate differential comes, as expected, from the fact that the debt coefficient changes sign at those longer horizons, as it goes from negative to positive. And while there is still a lot of noise and the individual debt coefficients are mostly marginally significant, the bottom right panel shows that the debt variables are jointly significant at the 5% level at almost all horizons.

7 Credit Spreads and Excess Currency Returns

In addition to the quantity based measure of the convenience yield presented in the previous section, I also examine the empirical relationship between excess currency returns and credit spreads in the data. The spread between the interest rate on Treasury notes and high quality corporate bonds is often used as a model-free measure of the convenience yield on Treasuries. The motivation is that corporate bonds are also a high quality debt instrument, but lack the exceptional liquidity and safety of Treasuries. As such, they provide lower non-pecuniary benefits, and investors require higher interest rates to hold them, making the corporate interest spread a proxy for the Treasury convenience yield. Thus, to the extent to which the US corporate spread proxies for the US convenience yield $\hat{\Psi}_t^H$, we would expect that foreign currency returns are positively correlated with the credit spread (as per equation 16).

To examine this in the data, I use the BAA-Treasury spread, since it is likely to capture both the safety and liquidity premia of Treasury notes (see Krishnamurthy and Vissing-Jorgensen (2012)). Clearly, this credit spread likely also includes a non-trivial default risk premium, which I control for by including two sets of default controls. First, I include the controls used by Krishnamurthy and Vissing-Jorgensen (2012) – the slope of the yield curve, measured as the spread between the 10-year and 3-month Treasury yields, and stock returns volatility, measured as the daily volatility of the S&P 500 over the last twelve months. And in addition, I also consider including the respective foreign counterparts of these controls for each currency pair. Hence, the full specification of the regression I estimate is

$$\lambda_{j,t+1} = \alpha_j + \beta(i_t - i^*_{j,t}) + \gamma(\text{BAA - Treasury}) + \text{Additional Controls} + \varepsilon_{j,t+1}$$

To further control for potential default premia effects, I exclude the Financial Crisis from the sample, because that was a period of extreme financial markets turbulence during which the BAA-Treasury spread was especially sensitive to risk-premia. Since I am hoping to use the spread as a proxy for the Treasury convenience yield, and not the default risk-premium, I stop the sample in Dec 31, 2007. Moreover, the data for the BAA interest rates starts on January 1, 1986 and hence the sample used for the regression above is 1986 - 2007.

The estimates are reported in Table 4. Columns (1) and (2) show results using the BAA-Treasury spread together with the default controls, and columns (3) and (4) include the interest rate differential as well. In all four specifications the coefficient on the BAA-Treasury spread is positive and significant, meaning that at times when the spreads are high, the return on foreign currency over the USD increases. This is consistent with the convenience yield mechanism, where the excess currency return compensates for the home convenience yield, and thus increases when it is high (i.e. when the credit spreads are high). Lastly, note that this relationship is unlikely to be driven by time-varying default risk – an increase in the default risk on USD assets would lead to a compensating increase in the excess currency return on the USD, not on the *foreign* currency as found here.

Table 4: Excess Currency Returns and Credit Spreads, 1986 - 2007

	(1)	(2)	(3)	(4)
BAA - Treasury	6.40**	5.51*	4.89*	4.77*
	(3.06)	(2.88)	(2.98)	(2.86)
$i_t - i_t^*$			-0.98***	-0.94***
			(0.33)	(0.37)
US Controls	Yes	Yes	Yes	Yes
Foreign Controls	No	Yes	No	Yes
Fixed Effects	Yes	Yes	Yes	Yes

Estimates with Driscoll and Kraay (1998) standard errors robust to heteroskedasticity, serial correlation and cross-equation correlation. ***, ** and * denote significance at the 1%, 5% and 10% level respectively.

I report the panel regression as the benchmark result, because the coefficient in pairwise regressions are consistently positive. In fact, the only pairwise negative coefficient, though never significant, occurs with the British pound. Excluding this outlier currency increases the significance of the credit spread coefficient across the board, but otherwise has no appreciable effect on the regression results. Interestingly, it is perhaps not surprising that the British pound is an outlier because the regression omits a measure of the foreign convenience yield (due to data limitations). Given that the pound is itself also a major international reserve currency, perhaps excluding measures of its convenience yield introduces a more significant omitted variable bias than in the case of the other currencies.

8 Conclusion

This paper proposes a new model of exchange rate determination that is consistent not only with the long standing classic UIP puzzle, but also with the more recent evidence that UIP violations reverse direction at longer horizons. This reversal has important implications about the underlying exchange rate behavior, implying that it follows a particular type of "delayed overshooting" characterized by excess depreciation at longer horizons. As argued by Engel (2016), the standard models of the puzzle are not consistent with this type of behavior.

Unlike previous models that have largely focused on time-varying risk and failure of rational expectations, my model relies on endogenous fluctuations in equilibrium bond convenience yields. In this model, excess currency returns (and hence UIP violations) arise as compensation for differences in the convenience yields between bonds denominated in different currencies. In particular, when the home convenience yield is low, both domestic interest rates and excess currency returns are high, as domestic and international investors require higher compensation to hold domestic debt.

This generates the classic UIP puzzle at short horizons that high interest rate currencies tend to earn high returns. The reversal in the direction of UIP violations at longer horizons is in turn tied to the interaction between monetary and fiscal policy. When monetary policy is independent, a sluggish tax policy introduces cyclical dynamics in government debt, which implies that UIP violations reverse direction at longer horizons. The explicit role played by the interaction of monetary and fiscal policy is an especially appealing feature of the model that is also borne out by the data. Lastly, I also empirically verify the key implications of the model that UIP violations are linked to debt dynamics. Overall, the model offers a rich new framework for international analysis that is also easily scalable.

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Online Appendix (For Online Publication)

A Data Description

The data set consists of forward and spot exchange rates from Reuters/WMR and Barclays, and is available on Datastream. It includes the Euro and the currencies of the following 18 advanced OECD countries: Australia, Austria, Belgium, Canada, Denmark, France, Germany, Ireland, Italy, Japan, the Netherlands, Norway, New Zealand, Portugal, Spain, Sweden, Switzerland and the UK.

The data spans the time period 1976:M1-2013:M6 and is at a daily frequency. The data on the Euro-legacy currencies (e.g. France, Austria, etc.), except for the German Deutsch Mark (DEM), ends in December 1998. As is common in the literature, instead of including separate DEM and EUR series, I combine the two by appending the Euro to the end of the DEM series. This creates a single long series that spans the whole time frame.

The data consists of forward and spot exchange rates, and I construct interest rate differentials from the Covered Interest Parity (CIP):

$$\frac{F_t}{S_t} = \frac{1+i_t}{1+i_t^*}$$

This is the standard practice in the literature because the data on forward contracts is better than data on short-term interest rates, since the forward market is deep and liquid.

B The UIP Condition

I define S_t to be the exchange rate, in terms of home currency per one unit of foreign currency (e.g. 1.25 USD per EUR), and i_t and i_t^* as the nominal interest rates on default-free bonds at home and abroad. For ease of exposition, I will refer to the US dollar as the "home" currency and the Euro as the "foreign" currency. A \$1 investment in US bonds at time t offers a return of $1+i_t$ dollars next period. The same \$1 invested in Euro denominated bonds would earn $\frac{S_{t+1}}{S_t}(1+i_t^*)$ dollars next period. First, we need to exchange this one dollar for Euros and obtain $\frac{1}{S_t}$ EUR in return. Investing this amount of Euros earns a gross interest rate of $1+i_t^*$ that next period can be exchanged back into dollars at the rate S_{t+1}, for a total return of $\frac{S_{t+1}}{S_t}(1+i_t^*)$ dollars.

Assuming that the law of one price holds, there exists a stochastic discount factor M_{t+1}, such that

$$E_t(M_{t+1}(1+i_t)) = 1 \tag{B.1}$$

$$E_t(M_{t+1}\frac{S_{t+1}}{S_t}(1+i_t^*)) = 1. \tag{B.2}$$

A straightforward way to obtain the Uncovered Interest Parity condition is to log-linearize the two equations, subtract them from one another and re-arrange to arrive at

$$E_t(s_{t+1} - s_t + i_t^* - i_t) = 0$$

where lower case letters represent variables in logs and I have used the approximation $i_t \approx \ln(1+i_t)$.[34] Thus, up to a first-order approximation, the expected return on foreign bonds, $E_t(s_{t+1} - s_t + i_t^*)$, equals the expected return on the home bond, i_t. This restricts the joint dynamics of exchange rates and interest rates, and delivers strong implications for exchange rate behavior. The condition obtains in a large class of standard open economy models.

B.1 The Classic UIP Puzzle

The failure of the UIP condition in the data is a long-standing and well documented puzzle in international finance, with a large and still active literature expanding on the seminal contributions by Bilson (1981) and Fama (1984). For excellent surveys, please see Hodrick (1987), Engel (1996, 2013).[35,36] The main finding is that there are *time-varying* excess returns in currency markets, and the puzzle is primarily about why there exist such volatile, and time-varying excess returns, and not necessarily simply why excess returns are not equalized.

Examining the UIP condition in the data is typically done by testing whether any variable in the time t information set can help forecast the return on foreign bonds relative to home bonds. As is standard in the literature I will equivalently refer to the relative return on foreign to home bonds as "excess return on foreign bonds" and also as "excess currency return". I denote the one period excess return from time t to $t+1$ as λ_{t+1}:

$$\lambda_{t+1} \equiv s_{t+1} - s_t + i_t^* - i_t.$$

The UIP condition requires $E_t(\lambda_{t+1}) = 0$ and hence $\text{Cov}(\lambda_{t+1}, X_t) = 0$ for any variable X_t in the time t information set. The vast majority of the literature focuses on some version of the original regression specification estimated by Fama (1984):

$$\lambda_{t+1} = \alpha_0 + \beta_1(i_t - i_t^*) + \varepsilon_{t+1} \tag{B.3}$$

where typically the base or "home" currency is the USD and i_t is the US interest rate. Under the null hypothesis that the UIP condition holds we should obtain $\alpha_0 = \beta_1 = 0$ so that the average excess return is zero and not forecastable by current interest rates. Contrary to this, numerous papers find that $\beta_1 < 0$ which implies that currencies which are experiencing high interest rates today are also expected to earn positive excess returns in the future.

I use monthly currency returns and interest rates to estimate regression (B.3). Since the underlying data is at the daily frequency, this creates overlapping periods in the dependent variable which induce serial correlation the error term. I correct for that by using Newey-

[34]The log-linearization is typically done around the symmetric steady state where $S_{t+1} = S_t = 1$ and $i_t = i_t^*$, because this allows us to express the condition in terms of the log-variables themselves. But the log-linearized condition holds for any arbitrary point of approximation.

[35] See also Canova (1991), Canova and Ito (1991), Bekaert and Hodrick (1992), Backus et al. (1993), Canova and Marrinan (1993), Cheng (1993), Hai et al. (1997), Bekaert (1995), Burnside (2013)

[36]A related, but not identical, finding is the high profitability of the carry trade, an investment strategy that goes long high-interest rate currencies and short low-interest rate currencies, and that should yield zero average return under UIP. Some papers that document profitable currency trading strategies are Lustig and Verdelhan (2007), Burnside et al. (2008), Brunnermeier et al. (2008), Burnside et al. (2010), Lustig et al. (2011), Menkhoff et al. (2012a)

West standard errors. The results are reported in Table B.1, and the estimates reaffirm the well established UIP Puzzle - I find that all β_1 point estimates are negative and almost all are statistically significant at conventional levels (15 out of 18). The evidence of negative and significant β_1 is remarkably consistent throughout all 18 currencies. Estimating equation (B.3) as a panel regression, where β_1 is restricted to be the same for all currency yields a significantly negative coefficient as well.

Table B.1: UIP Regression Currency by Currency

Country	Currency	α_0	(s.e.)	β_1	(s.e.)	$\chi^2(\alpha_0 = \beta_1 = 0)$	R^2
Australia	AUD	-0.001	(0.002)	-1.63***	(0.48)	16.3***	0.014
Austria	ATS	0.002	(0.002)	-1.75***	(0.58)	9.5***	0.023
Belgium	BEF	-0.0002	(0.002)	-1.58***	(0.39)	17.5***	0.025
Canada	CAD	-0.003	(0.001)	-1.43***	(0.38)	19.1***	0.013
Denmark	DKK	-0.001	(0.001)	-1.51***	(0.32)	25.4***	0.025
France	FRF	-0.001	(0.002)	-0.84	(0.63)	1.9	0.007
Germany	DEM	0.002	(0.001)	-1.58***	(0.57)	7.9**	0.015
Ireland	IEP	-0.002	(0.002)	-1.32***	(0.38)	12.3***	0.020
Italy	ITL	-0.002	(0.002)	-0.79**	(0.33)	7.0**	0.013
Japan	JPY	0.006***	(0.002)	-2.76***	(0.51)	28.9***	0.038
Netherlands	NLG	0.003	(0.002)	-2.34***	(0.59)	16.0***	0.041
Norway	NOK	-0.0003	(0.001)	-1.15***	(0.39)	10.4***	0.013
New Zealand	NZD	-0.001	(0.002)	-1.74***	(0.39)	28.3***	0.038
Portugal	PTE	-0.002	(0.002)	-0.45**	(0.20)	5.9*	0.019
Spain	ESP	0.002	(0.003)	-0.19	(0.46)	2.8	0.001
Sweden	SEK	0.0001	(0.001)	-0.42	(0.50)	0.9	0.002
Switzerland	CHF	0.005***	(0.002)	-2.06***	(0.55)	13.9***	0.026
UK	GBP	-0.003**	(0.001)	-2.24***	(0.60)	14.2***	0.028
Panel, pooled		0.0002	(0.001)	-0.79***	(0.15)	22.3***	
Panel, fixed eff.				-1.01***	(0.21)	19.1***	

This table presents estimates of α_0 and β_1 from the regression $s_{j,t+1} - s_{j,t} + i^*_{j,t} - i_{j,t} = \alpha_{j,0} + \beta_{j,1}(i_{j,t} - i^*_{j,t}) + \varepsilon_{j,t+1}$. The standard errors in single currency regressions are Newey-West errors robust to serial correlation. The standard errors for the panel estimations are computed according to the Driscoll and Kraay (1998) method that is robust to heteroskedasticity, serial correlation and contemporaneous correlation across equations. The base currency is the USD.

C Proofs

C.1 LEMMA 1:

LEMMA 1 (Existence and Uniqueness). *A determinate stationary equilibrium exists if and only if we have one of the following two policy combinations:*

(i) Active Monetary, Passive Fiscal policy: $\phi_\pi > 1$, $\kappa_b \in (\theta - \theta_2, \frac{1+\rho_\tau}{1-\rho_\tau}(\theta + \theta_2))$, $\rho_\tau \in [0, \frac{\theta_2}{\theta})$.

(ii) Passive Monetary, Active Fiscal policy: $\phi_\pi < 1$, $\kappa_b \notin (\theta - \theta_2, \frac{1+\rho_\tau}{1-\rho_\tau}(\theta + \theta_2))$, $\rho_\tau \in [0, 1)$.

where $\theta > \theta_2 \geq 1$, *with* $\theta = (1+i)(1+\gamma_\Psi + \gamma_M)$, $\theta_2 = 1 + \gamma_M(1+i)$, $\gamma_\Psi > 0$, *and* $\gamma_M \geq 0$.

Proof. I will first show the if direction. The equilibrium of the model is described by four (log-linearized) equations: Euler equation for home bonds, government budget, the Taylor rule and the tax rule. These equations determine the dynamics of the four domestic equilibrium variables – inflation, interest rates, government debt and taxes – and represent a closed system that can be solved independent of foreign variables considerations.

Using the fact that consumption and foreign bonds holdings are constant, the the log-linearized MRS becomes,

$$\hat{M}_{t+1} = \gamma_M(\hat{b}_{h,t+1} - \hat{b}_{h,t})$$

where $\gamma_M = \frac{u_{cb_h}(c,b_h,b_f)}{u_c(c,b_h,b_f)} b_h > 0$, and the log-linearized convenience benefit is:

$$\frac{\Psi^H}{\beta(1+i)}\hat{\Psi}^H = -\gamma_\Psi \hat{b}_{ht}$$

where $\gamma_\Psi = -\frac{b_h}{\beta(1+i)}\frac{1}{u_c(c,b_h,b_f)}(u_{b_h b_h}(c,b_h,b_f) - u_{b_h}(c,b_h,b_f)\frac{u_{cb_h}(c,b_h,b_f)}{u_c(c,b_h,b_f)}) > 0$. I am using the convention that $u_x(.)$ represents the partial derivative in respect to x, and $u_{xx}(.)$ represents the second partial and so on. Variables without time subscripts are steady-state values.

Using these relationships, and the fact that in equilibrium home agent bond holdings equal the supply of home government debt, the system of equilibrium conditions becomes

$$\begin{aligned}
\hat{i}_t &= E_t(\hat{\pi}_{t+1}) + \gamma_\Psi \hat{b}_t - \gamma_M(E_t(\hat{b}_{h,t+1}) - \hat{b}_{h,t}) \\
\hat{b}_{ht} + \frac{\tau}{b_h}\hat{\tau}_t &= (1+i)(\hat{b}_{h,t-1} + \hat{i}_{t-1} - \hat{\pi}_t) \\
\hat{i}_t &= \phi_\pi \hat{\pi}_t + v_t \\
\hat{\tau}_t &= \rho_\tau \hat{\tau}_{t-1} + (1-\rho_\tau)\kappa_b \frac{b_h}{\tau}\hat{b}_{h,t-1}
\end{aligned}$$

First, I show that condition (i), Active monetary/passive fiscal policy mix, ensures that a determinate, stable equilibrium exists. Assume that $\phi_\pi > 1$, $\kappa_b \in (\theta - \theta_2, \frac{1+\rho_\tau}{1-\rho_\tau}(\theta + \theta_2))$, and $\rho_\tau \in [0, \frac{\theta_2}{\theta})$, where $\theta = (1+i)(1+\gamma_\Psi + \gamma_M)$, and $\theta_2 = 1 + (1+i)\gamma_M$. Substituting the Taylor rule into the Euler equation for the home bonds, and solving forward for inflation:

$$\hat{\pi}_t = \frac{1}{\phi_\pi}\Big(E_t(\hat{\pi}_{t+1}) + (\gamma_\Psi + \gamma_M)\hat{b}_t - \gamma_M E_t(\hat{b}_{h,t+1}) - v_t\Big)$$
$$\vdots$$
$$= \frac{\gamma_M}{\phi_\pi}\hat{b}_{ht} - \frac{v_t}{\phi_\pi} + \frac{\gamma_\Psi + \gamma_M(1-\phi_\pi)}{\phi_\pi}\sum_{j=0}^{\infty}\frac{1}{\phi_\pi^j}E_t(\hat{b}_{h,t+j})$$

Next, date the government budget constraint one period forward, take an expectation conditional on time t information and use the Euler equation and the tax rule to substitute out the interest rate and inflation, and arrive at the following 2 equations:

$$\underbrace{E_t\begin{bmatrix}\hat{b}_{h,t+1}\\ \hat{\tau}_{t+1}\end{bmatrix}}_{x_{t+1}} = \underbrace{\begin{bmatrix}\frac{\theta-(1-\rho_\tau)\kappa_b}{\theta_2} & -\frac{\tau}{b}\frac{\rho_\tau}{\theta_2}\\ (1-\rho_\tau)\kappa_b\frac{b}{\tau} & \rho_\tau\end{bmatrix}}_{A}\underbrace{\begin{bmatrix}\hat{b}_{ht}\\ \hat{\tau}_t\end{bmatrix}}_{x_t} \tag{C.1}$$

I will show that condition (i) ensures that the eigenvalues of the auto-regressive matrix A are inside the unit circle, and hence we can use this system to solve for the infinite sum of expected b_{ht} in the expression for equilibrium inflation. The two eigenvalues of A are

$$\lambda_{1,2} = \frac{\theta-(1-\rho_\tau)\kappa_b+\theta_2\rho_\tau \pm \sqrt{(\theta-(1-\rho_\tau)\kappa_b+\theta_2\rho_\tau)^2 - 4\theta\theta_2\rho_\tau}}{2\theta_2}.$$

The eigenvalues are complex conjugates when $(\theta-(1-\rho_\tau)\kappa_b+\theta_2\rho_\tau)^2 - 4\theta\theta_2\rho_\tau < 0$. The left-hand side of this equation defines a quadratic expression in ρ_τ that is convex and crosses zero at the following two points

$$\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b-\theta)+(\kappa_b+\theta)\theta_2 - 2\sqrt{\kappa_b\theta\theta_2(\kappa_b-\theta+\theta_2)}}{(\theta_2+\kappa_b)^2}$$

$$\overline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b-\theta)+(\kappa_b+\theta)\theta_2 + 2\sqrt{\kappa_b\theta\theta_2(\kappa_b-\theta+\theta_2)}}{(\theta_2+\kappa_b)^2}$$

Since $\theta_2 < \theta$ it follows that $\underline{\rho}(\kappa_b) < 1$ and since

$$\kappa_b(\kappa_b-\theta)+(\kappa_b+\theta)\theta_2 = \kappa_b(\kappa_b-\theta+\theta_2)+\theta\theta_2$$

it follows that $\underline{\rho}(\kappa_b) > 0$. Moreover, $\underline{\rho}(\kappa_b) \le \frac{\theta_2}{\theta} \le \overline{\rho}_\tau(\kappa_b)$, and hence for $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ the eigenvalues are real, and for $\rho \in (\underline{\rho}(\kappa_b), \frac{\theta_2}{\theta})$ they are complex conjugates.

First, I address the case where the eigenvalues are complex. Their magnitude its:

$$
\begin{aligned}
|\lambda_k| &= \frac{1}{2\theta_2}\left((\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)^2 + [4\theta\theta_2\rho_\tau - (\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)^2]\right)^{\frac{1}{2}} \\
&= \frac{1}{2\theta_2}\sqrt{4\theta\theta_2\rho_\tau} \\
&= \sqrt{\frac{\theta}{\theta_2}\rho_\tau}
\end{aligned}
$$

and hence $|\lambda_k| < 1$ if and only if $\rho_\tau < \frac{\theta_2}{\theta}$. This is satisfied by condition (i), and hence when the eigenvalues are complex, they lie inside the unit circle.

Next, I address the situation when the eigenvalues are real, $\rho_\tau < \underline{\rho}_\tau(\kappa_b)$. First, I will show that $\kappa_b = \theta - \theta_2$ is the minimum value for which the eigenvalues are both inside the unit circle. For $\kappa_b = \theta - \theta_2$ we have $\underline{\rho}(\kappa_b) = \overline{\rho}(\kappa_b) = \frac{\theta_2}{\theta}$, and hence the roots are real for all values of ρ_τ under condition (i). Moreover, for that value of κ_b:

$$
\begin{aligned}
\lambda_1 &= \frac{1}{2\theta_2}(\theta_2 + \rho_\tau\theta + \sqrt{(\theta_2 + \rho_\tau\theta)^2 - 4\theta\theta_2\rho_\tau}) \\
&= \frac{1}{2\theta_2}(\theta_2 + \rho_\tau\theta + \sqrt{(\theta_2 - \rho_\tau\theta)^2}) \\
&= 1
\end{aligned}
$$

while $\lambda_2 = \rho_\tau\frac{\theta}{\theta_2} < 1$. Next, notice that when $\kappa_b < \frac{\theta+\theta_2\rho_\tau}{1-\rho_\tau}$ we have $\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau > 0$ and thus $\lambda_1 > 0$ whenever it is real. Furthermore,

$$
\frac{\partial\lambda_1}{\partial\kappa_b} = -\frac{1-\rho_\tau}{2\theta_2} - \frac{(1-\rho_\tau)(\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)}{2\theta_2\sqrt{(\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)^2 - 4\theta\theta_2\rho_\tau}} < 0
$$

and hence for $\kappa_b \in (\theta - \theta_2, \frac{\theta+\theta_2\rho_\tau}{1-\rho_\tau})$ we have $\lambda_1 \in (0,1)$. Moreover, for those values of κ_b $\lambda_2 > 0$ as well (when real), and since whenever the eigenvalues are real $(\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)^2 - 4\theta\theta_2\rho_\tau \geq 0$ and thus $\lambda_2 < \lambda_1$, it follows that

$$
0 < \lambda_2 < \lambda_1 < 1
$$

for all $\kappa_b \in (\theta - \theta_2, \frac{\theta+\theta_2\rho_\tau}{1-\rho_\tau})$.

On the other hand, if $\kappa_b = \frac{\theta+\theta_2\rho_\tau}{1-\rho_\tau}$, then the eigenvalues are complex for all $\rho_\tau > 0$, and when $\rho_\tau = 0$, then $\lambda_1 = \lambda_2 = 0$.

Lastly, consider $\kappa_b \in (\frac{\theta+\theta_2\rho_\tau}{1-\rho_\tau}, \frac{(\theta+\theta_2)(1+\rho_\tau)}{1-\rho_\tau})$. In this case, whenever the eigenvalues are real they are negative since

$$
\begin{aligned}
\lambda_1 &= \frac{\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau + \sqrt{(\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)^2 - 4\theta\theta_2\rho_\tau}}{2} \\
&\leq \frac{\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau + |\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau|}{2} \\
&\leq 0
\end{aligned}
$$

and thus $\lambda_2 \leq \lambda_1 \leq 0$. Furthermore,

$$
\frac{\partial \lambda_2}{\partial \kappa_b} = -\frac{1-\rho_\tau}{2\theta_2} + \frac{(1-\rho_\tau)(\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)}{2\theta_2\sqrt{(\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau)^2 - 4\theta\theta_2\rho_\tau}} < 0
$$

since $\theta - (1-\rho_\tau)\kappa_b + \theta_2\rho_\tau < 0$, and at $\kappa_b = \frac{(\theta+\theta_2)(1+\rho_\tau)}{1-\rho_\tau}$ we have

$$
\lambda_2 = -1
$$

Therefore, for $\kappa_b \in (\theta - \theta_2, \frac{1+\rho_\tau}{1-\rho_\tau}(\theta+\theta_2))$ and $\rho_\tau < \underline{\rho}_\tau(\kappa_b)$ the eigenvalues are real and less than 1 in absolute value. And as we have already shown, since $\rho_\tau < \frac{\theta_2}{\theta}$, whenever the eigenvalues are complex they are also less than 1 in modulus.

Thus, condition (i) implies that the eigenvalues of A lie inside the unit circle, so then

$$
\sum_{j=0}^{\infty} \frac{1}{\phi_\pi^j} E_t(\hat{b}_{h,t+j}) = [1,0] * (I - \frac{1}{\phi_\pi}A)^{-1} \begin{bmatrix} \hat{b}_{ht} \\ \hat{\tau}_t \end{bmatrix}
$$

and we can use this expression to solve for equilibrium inflation in terms of debt and taxes at time t. We can then substitute the interest rate and inflation, and arrive at a 2 equation system that determines $\hat{b}_{ht}$ and $\hat{\tau}_t$:

$$
\begin{bmatrix} \hat{b}_{h,t+1} \\ \hat{\tau}_{t+1} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\theta-(1-\rho_\tau)\kappa_b}{\theta_2} & -\frac{\tau}{b}\frac{\rho_\tau}{\theta_2} \\ (1-\rho_\tau)\kappa_b\frac{b}{\tau} & \rho_\tau \end{bmatrix}}_{=A} \begin{bmatrix} \hat{b}_{ht} \\ \hat{\tau}_t \end{bmatrix} + \underbrace{\begin{bmatrix} \frac{1+i}{\phi_\pi} \\ 0 \end{bmatrix}}_{B} v_t \tag{C.2}
$$

Unsurprisingly, the auto-regressive matrix is the same matrix A we have already analyzed. As a result, we know that when condition (i) holds, its eigenvalues are inside the unit circle and we have a stationary solution for debt and taxes.

Now assume that condition (ii) holds so $\phi_\pi < 1$, $\kappa_b \notin (\theta - \theta_2, (\theta+\theta_2)\frac{1+\rho_\tau}{1-\rho_\tau})$, and $\rho_\tau < 1$. In this case we cannot solve for inflation forward, however, equation (C.1) still holds and now I will show that $\kappa_b \notin (\theta - \theta_2, (\theta+\theta_2)\frac{1+\rho_\tau}{1-\rho_\tau})$ implies that at least one of the eigenvalues of A is greater than 1 in absolute value.

First, note that for $\kappa_b < \theta - \theta_2$

$$
(\theta - (1-\rho_\tau)\kappa_b + \rho_\tau\theta_2)^2 - 4\theta\theta_2\rho_\tau \geq 0
$$

and hence the eigenvalues are always real. Moreover, above we showed that when the eigenvalues are real, $\frac{\partial\lambda_1}{\partial\kappa_b} < 0$ and that $\lambda_1 = 1$ when $\kappa_b = \theta - \theta_2$, hence it follows that $\lambda_1 > 1$

for any $\kappa_b < \theta - \theta_2$. Similarly, if $\kappa_b > (\theta + \theta_2)\frac{1+\rho_\tau}{1-\rho_\tau}$, the roots are also always real and as we have shown above at $\kappa_b = (\theta + \theta_2)\frac{1+\rho_\tau}{1-\rho_\tau}$, $\lambda_2 = -1$ and it is decreasing in κ_b. So it follows that for $\kappa_b > (\theta + \theta_2)\frac{1+\rho_\tau}{1-\rho_\tau}$, we have $\lambda_2 < -1$, and in either case we have an eigenvalue greater than one.

If A is diagonalizable, we can express equation (C.1) as

$$E_t(x_{t+1}) = P\Lambda P^{-1}x_t$$

where $x_t = \begin{bmatrix} \hat{b}_{h,t} \\ \hat{\tau}_t \end{bmatrix}$, and Λ is a diagonal matrix with the eigenvalues of A on the diagonal, and P is the matrix of corresponding eigenvectors. We can then multiply on both sides by P^{-1}, define $\tilde{x}_t = P^{-1}x_t$ and obtain the diagonal system

$$E_t(\tilde{x}_{t+1}) = \Lambda\tilde{x}_t$$

and in particular,

$$E_t(\tilde{x}^{(1)}_{t+1}) = \lambda_1\tilde{x}^{(1)}_t \tag{C.3}$$

where $\tilde{x}^{(1)}_t$ is the first element of the vector. If A is not diagonalizable, then we can use the Jordan Normal form where P is the matrix of generalized eigenvalues, and Λ is upper triangular, with the repeated eigenvalue on the diagonal, and 1 in the upper right corner. We can then use the second equation of the resulting system to arrive at a univariate equation similar to (C.3) where the repeated eigenvalue $|\lambda| > 1$ is the coefficient. Everything else then follows in the same manner.

We can then solve (C.3) forward (since $|\lambda_1| > 0$) and obtain

$$\tilde{x}^{(1)}_{t+1} = \lim_{j\to\infty}\frac{1}{\lambda_1^j}E_t(\tilde{x}^{(1)}_{t+j}) = 0$$

Recall that $\tilde{x}_t = P^{-1}x_t$ and hence a linear combination of $\hat{b}_{ht}$ and $\hat{\tau}_t$ is equal to 0, therefore we can write

$$\hat{\tau}_t = K\hat{b}_t$$

for some constant K. Substituting in the tax rule equation for debt, we obtain

$$\hat{\tau}_t = (\rho_\tau - (1-\rho_\tau)\kappa_b\frac{b_h}{\tau}K)\hat{\tau}_{t-1}$$

which implies that the solution is

$$\hat{\tau}_t = \hat{b}_{ht} = 0$$

Next, we can substitute this result in the government budget and obtain the relationship

$$i_{t-1} = \pi_t$$

Substituting in the Taylor rule we find the solution for inflation:

$$\pi_t = \phi_\pi \pi_{t-1} + v_{t-1}$$

Since $\phi_\pi < 1$, this is stationary and this concludes the forward direction of the proof. We have shown that when either conditions (i) or (ii) are satisfied, there is a determinate stationary equilibrium.

In proving the necessary direction, I start with the case where $\phi_\pi > 1$. This time I will first deal with the conditions on κ_b, and to this end assume that $\kappa_b < \theta - \theta_2$. Above we showed that in this case the roots are always real, and that $\lambda_1\Big|_{\kappa_b=\theta-\theta_2} = 1$, and that $\frac{\partial\lambda_1}{\partial\kappa_b} < 0$ for $\kappa < \frac{\theta+\theta_2}{1-\rho_\tau}$ which holds since $\theta - \theta_2 < \frac{\theta+\theta_2}{1-\rho_\tau}$. Therefore, it is immediate that $\kappa_b < \theta - \theta_2$ leads to a root bigger than one and thus explosive solutions.

On the other hand if $\kappa_b > \frac{(\theta+\theta_2)(1+\rho_\tau)}{1-\rho_\tau}$, then

$$(\theta - (1-\rho_\tau)\kappa_b + \rho_\tau\theta_2)^2 - 4\theta\theta_2\rho_\tau \geq 0$$

so the roots are again always real. Moreover, we have already shown that $\lambda_2\Big|_{\kappa_b=\frac{(\theta-\theta_2)(1+\rho_\tau)}{1-\rho_\tau}} = -1$, and that $\frac{\partial\lambda_2}{\partial\kappa_b} < 0$ for $\kappa_b > \frac{(\theta-\theta_2)(1+\rho_{tau})}{1-\rho_\tau}$, hence it follows that $|\lambda_2| > 1$ for all $\kappa_b > \frac{(\theta-\theta_2)(1+\rho_\tau)}{1-\rho_\tau}$, and thus we again have an explosive root.

Next, turn attention to $\rho_\tau > \frac{\theta_2}{\theta}$ and $\kappa_b \in (\theta-\theta_2, \frac{(\theta-\theta_2)(1+\rho_{tau})}{1-\rho_\tau})$. If $\rho_\tau \in [\frac{\theta_2}{\theta}, \overline{\rho}_\tau(\kappa_b))$ then the resulting complex eigenvalues will be outside of the unit circle and there are no non-explosive solutions for debt and taxes. On the other hand, if $\rho_\tau \geq \overline{\rho}_\tau(\kappa_b)$, then

$$\frac{\partial\lambda_1}{\partial\rho_\tau} = \frac{\kappa_b+\theta_2}{2\theta_2} + \frac{1}{2\theta_2}\frac{(\kappa_b+\theta_2)(\theta-(1-\rho_\tau)\kappa_b+\rho_\tau\theta_2) - 2\theta\theta_2}{\sqrt{(\theta-(1-\rho_\tau)\kappa_b+\rho_\tau\theta_2)^2 - 4\theta\theta_2\rho_\tau}} > 0$$

since $\kappa_b + \theta_2 > \theta > 1$ and $(\theta - (1-\rho_\tau)\kappa_b + \rho_\tau\theta_2) - 2\theta\theta_2\rho_\tau \geq 0$. Moreover,

$$\begin{aligned}
\lambda_1\Big|_{\rho_\tau=\overline{\rho}_\tau(\kappa_b)} &= \frac{\theta + \sqrt{\kappa_b\frac{\theta}{\theta_2}(\kappa_b - (\theta-\theta_2))}}{\kappa_b+\theta_2} \\
&> \frac{\theta + (\kappa_b - (\theta-\theta_2))}{\kappa_b+\theta_2} \\
&= 1
\end{aligned}$$

where the inequality follows from the fact that $\theta > \theta_2$, and hence $\kappa_b\frac{\theta}{\theta_2} > \kappa_b > \kappa_b - (\theta-\theta_2)$. Thus, we see that $\lambda_1 > 1$ and hence we again have an explosive root.

Next, I treat the case $\phi_\pi < 1$. If $\kappa_b \in [\theta-\theta_2, \frac{(\theta-\theta_2)(1+\rho_\tau)}{1-\rho_\tau}]$, then either the autoregressive matrix A has a unit root (unstable solutions), or it has both eigenvalues inside the unit circle. When both roots are inside the unit circle, then conditional on a process for equilibrium inflation, we can solve for debt and taxes backwards. However, in this case we do not have a determinate solution for inflation – in fact there could be many inflation processes that would satisfy the government budget constraint and the Euler equations for bonds. To see this, you let ε^{π}_{t+1} be the expectational error defined as

$$\hat{\pi}_{t+1} = E_t(\hat{\pi}_{t+1}) + \varepsilon^{\pi}_{t+1}$$

Using this expression we can again reduce to a system of 2 equations that define a first-order difference system for $\hat{b}_{ht}$ and $\hat{\tau}_t$, with A as the auto-regressive matrix. That defines stationary solutions for debt and taxes, conditional on the expectational error ε^{π}_{t+1}. Then, we can substitute the Taylor rule in the Euler equation and arrive at

$$\pi_{t+1} = \phi_\pi \hat{\pi}_t + v_t - (\gamma_\Psi + \gamma_M)\hat{b}_{ht} + \gamma_M E_t(\hat{b}_{h,t+1}) - \varepsilon^{\pi}_{t+1}$$

Since $\phi_\pi < 1$ and $\hat{b}_{ht}$ is stationary, this defines a stationary process for equilibrium inflation. However, the expectational error ε^{π}_{t+1} is undetermined, and as a result many different processes for inflation satisfy the equilibrium conditions. Thus, with $\phi_\pi < 1$ and $\kappa_b \in (\theta - \theta_2, \frac{(\theta-\theta_2)(1+\rho_{tau})}{1-\rho_\tau})$ the equilibrium is indeterminate. □

C.2 LEMMA 2:

LEMMA 2 (IRF: Active Monetary/Passive Fiscal). *Let $\phi_\pi > 1$, $\kappa_b \in (\theta-\theta_2, \frac{\theta+(\theta_2-1)\rho_\tau}{1-\rho_\tau})$, and define $\underline{\rho}(\kappa_b) = \frac{\kappa_b(\kappa_b+\theta_2-\theta)+\theta\theta_2-2\sqrt{\kappa_b\theta\theta_2(\kappa_b+\theta_2-\theta)}}{(\theta_2+\kappa_b)^2} > 0$. Then,*

(i) If $\rho_\tau \in [0, \underline{\rho}(\kappa_b)]$ the matrix A in (12) has two real, positive eigenvalues, and thus the IRF is positive and declines to zero monotonically:

$$a_{bk} > 0 \text{ for } k = 0, 1, 2, 3, \ldots$$

(ii) If $\rho_\tau \in (\underline{\rho}(\kappa_b), \frac{\theta_2}{\theta})$ the matrix A in (12) has a pair of complex conjugate eigenvalues, $\lambda = a \pm bi$, and conjugate eigenvectors $\vec{v}_k = [x \pm yi, 1]'$, where a, b, x, y are real numbers and i is the imaginary unit. Thus, the IRF follows the dampened cosine wave:

$$a_{bk} = |\lambda|^k \sqrt{1 + (\frac{x}{y})^2} \cos(k\zeta + \psi - \frac{\pi}{2}), \text{ for } k = 1, 2, 3, \ldots$$

where $\zeta = \arctan(\frac{b}{a})$, $\psi = \arctan(\frac{y}{x})$ and $a_{bk} > 0$ for $k \in \{0, 1\}$.

Proof. **Part (i):** The first part follows directly from the proof of Lemma 1 – $\rho_\tau \leq \underline{\rho}_\tau(\kappa_b)$ ensures that the eigenvalues are real, and $\kappa_b \in (\theta - \theta_2, \frac{\theta+(\theta_2-1)\rho_\tau}{1-\rho_\tau})$ ensures they are both positive.

To characterize the IRF note that the Wold decomposition of x_t is

$$x_t = Bv_t + ABv_{t-1} + A^2 Bv_{t-2} + \ldots$$

and use the fact that

$$B = \begin{bmatrix} \frac{1+i}{\phi_\pi} \\ 0 \end{bmatrix} v_t$$

to obtain

$$\hat{b}_{ht} = \frac{1+i}{\phi_\pi}(v_t + a_{11}^{(1)}v_{t-1} + a_{11}^{(2)}v_{t-2} + a_{11}^{(3)}v_{t-3} + \dots)$$
$$\hat{\tau}_t = \frac{1+i}{\phi_\pi}(a_{21}^{(1)}v_{t-1} + a_{21}^{(2)}v_{t-2} + a_{21}^{(3)}v_{t-3} + \dots)$$

where $a_{lm}^{(k)}$ is the (l,m) element of the matrix A^k. Define $a_{11}^{(0)} = 1$ and $a_{21}^{(0)} = 0$ and the transformation

$$a_{bk} = \frac{1+i}{\phi_\pi}a_{11}^{(k)}.$$

The sequence $\{a_{bk}\}_{k=0}^{\infty}$ defines the Impulse Response Functions of $\hat{b}_{ht}$.

First, I will show that $a_{bk} \geq 0$ for all $k = 0, 1, 2, \dots$ when the matrix A is diagonalizable, and then I will handle the case when the eigenvalue is repeated and A is not diagonalizable (the only other case we need to worry about for a two by two matrix).

Assuming that A is diagonalizable, define

$$\Lambda = \begin{bmatrix} \lambda_1 & 0 \\ 0 & \lambda_2 \end{bmatrix}$$

as a matrix with the two eigenvalues of A on its diagonal ordered like $\lambda_1 > \lambda_2$ (remember we are handling the case of real eigenvalues right now) and P as a matrix that has the eigenvectors of A as its columns. Since we have assumed A is diagonalizable, we have $A = P\Lambda P^{-1}$ and also $A^k = P\Lambda^k P^{-1}$. Since Λ is diagonal

$$\Lambda^k = \begin{bmatrix} \lambda_1^k & 0 \\ 0 & \lambda_2^k \end{bmatrix}$$

and thus if we expand the expression for A^k we obtain that

$$a_{11}^{(k)} = \frac{p_{11}p_{22}\lambda_1^k - p_{12}p_{21}\lambda_2^k}{|P|}$$

where $|P|$ is the determinant of the matrix of eigenvectors P and p_{lm} is its (l,m)-th element. Since both of the eigenvalues are positive and are ordered so that $\lambda_1 > \lambda_2$ it follows that $|P| > 0$ and hence

$$\frac{p_{11}p_{22}\lambda_1^k - p_{12}p_{21}\lambda_2^k}{|P|} > 0.$$

This proves that $a_{11}^{(k)} > 0$ for all k and hence $a_{bk} > 0$ for all k. This completes the proof for diagonalizable A – now assume that A is not diagonalizable. We can instead use the Jordan Decomposition to again write $A = P\Lambda P^{-1}$ but now

$$\Lambda = \begin{bmatrix} \lambda & 1 \\ 0 & \lambda \end{bmatrix}$$

and the columns of P are the generalized eigenvectors of A. In this case, there is only one linearly independent eigenvector associated with the eigenvalue of λ, call it $\vec{p}$, and thus the second generalized eigenvector, call it $\vec{u}$, is a 2x1 vector that solves

$$(A - \lambda I)\vec{u} = \vec{p}$$

We can solve for the needed eigenvectors via standard techniques, and obtain $\vec{p} = [p_1, 1]'$ and $\vec{u} = [u_1, 1]'$, where $p_1 = \frac{\lambda - \rho_\tau}{(1-\rho_\tau)\kappa_b \frac{\tau}{b_h}}$, $u_1 = p_1 + \frac{1}{(1-\rho_\tau)\kappa_b \frac{\tau}{b_h}}$. We can then use $A^k = P\Lambda^k P^{-1}$ to get:

$$a_{11}^{(k)} = \lambda^{k-1}(\lambda + k\frac{p_1}{u_1 - p_1}) > 0$$

The inequality follows from $u_1 > p_1 > 0$, $\lambda > 0$. This completes the proof of part (i).

Part (ii): From the proof of Lemma 1 we know that $\rho_\tau \in (\underline{\rho}_\tau, \frac{\theta_2}{\theta})$ implies that the eigenvalues of A are complex. We can express them as $\lambda_1 = a + bi$ and $\lambda_2 = a - bi$ where $a = \frac{1}{2}(\theta - (1-\rho_\tau)\kappa_b + \rho_\tau\theta_2) > 0$, $b = \frac{1}{2}\sqrt{4\theta\theta_2\rho_\tau - (\theta - (1-\rho_\tau)\kappa_b + \rho_\tau\theta_2)^2} > 0$ and i is the imaginary unit. The two conjugate eigenvectors can be written as $\vec{p}_k = [x \pm yi, 1]'$, where .

$$x = \frac{\tau}{b_h}\frac{(\theta - (1-\rho_\tau)\kappa_b + \rho_\tau\theta_2 - 2\rho_\tau)}{2(1-\rho_\tau)\kappa_b}$$

$$y = \frac{\tau}{b_h}\frac{\sqrt{4\theta\theta_2\rho_\tau - (\theta - (1-\rho_\tau)\kappa_b + \rho_\tau\theta_2)^2}}{2b(1-\rho_\tau)\kappa_b}$$

With two conjugate complex eigenvalues A is diagonalizable and can be expressed as $A = P\Lambda P^{-1}$ where P is a similarity matrix with the eigenvectors of A as its columns and Λ is a diagonal matrix with the eigenvalues on the diagonal. By Euler's formula $\lambda_1 = a + bi = |\lambda|e^{\zeta i}$ where $\zeta = \arctan(\frac{b}{a})$ and $|\lambda| = \sqrt{a^2 + b^2}$ is the magnitude of the complex roots. This formulation is convenient because it is easy to take powers of the eigenvalues, (e.g. $\lambda_1^k = |\lambda|^k e^{k\zeta i}$) and hence it is easy to compute powers of the eigenvalue matrix Λ. Using this, Euler's formula and the fact that $A^k = P\Lambda^k P^{-1}$ it is straightforward to compute

$$\begin{aligned} a_{11}^{(k)} &= |\lambda|^k(\cos(k\zeta) + \frac{x}{y}\sin(k\zeta)) \\ &= |\lambda|^k\sqrt{1 + (\frac{x}{y})^2}\sin(k\zeta + \psi) \\ &= |\lambda|^k\sqrt{1 + (\frac{x}{y})^2}\cos(k\zeta + \psi - \frac{\pi}{2}) \end{aligned}$$

where $\psi = \arctan(\frac{y}{x}) + \pi\mathbb{I}(\frac{y}{x} < 0)$. The second equality follows from the formula for linear combinations of trig functions, and the third is simply an application of $\cos(\theta - \frac{\pi}{2}) = \sin(\theta)$.

By the definition of the $\arctan(\cdot)$ function and the virtue of $a \geq 0, b \geq 0$ it follows that $\zeta \in [0, \frac{\pi}{2})$. If $\kappa_b \leq \frac{\theta+(\theta_2-2)\rho_\tau}{1-\rho_\tau}$, then $x \geq 0$ and $\psi \leq \frac{\pi}{2}$ and this case $\cos(k\zeta + \psi - \frac{\pi}{2}) \geq 0$ for at least $k = 1$. Otherwise, use the formula for addition of arctangent to get,

$$\arctan(\frac{b}{a}) + \arctan(\frac{y}{x}) = \arctan(\frac{\frac{b}{a} + \frac{y}{x}}{1 - \frac{by}{ax}}).$$

where $1 - \frac{by}{ax} > 0$. And since $\kappa_b \in (\frac{\theta+(\theta_2-2)\rho_\tau}{1-\rho_\tau}, \frac{\theta+(\theta_2-1)\rho_\tau}{1-\rho_\tau})$, we can show that $\frac{b}{a} + \frac{y}{x} < 0$ and therefore $\arctan(\frac{\frac{b}{a}+\frac{y}{x}}{1-\frac{by}{ax}}) \in (-\frac{\pi}{2}, 0)$. Therefore, we again reach the conclusion that $\cos(k\zeta + \psi - \frac{\pi}{2}) \geq 0$ for at least $k = 1$. This completes the proof of Lemma 2. □

C.3 LEMMA 3:

LEMMA 3 (IRF: Passive Monetary/Active Fiscal). *Let $\phi_\pi < 1$, $\kappa_b \in [0, \theta - \theta_2)$, $\rho_\tau \in [0, 1)$. Then, the system has two real, positive eigenvalues for all $\rho_\tau \in [0, 1)$, and thus the IRF does not cross steady state. Moreover, debt is in fact constant:*

$$a_{bk} = 0 \textit{ for } k = 0, 1, 2, 3, \ldots$$

Proof. From the proof of Lemma 1 we know that $\kappa_b < \theta - \theta_2$ ensures the eigenvalues are real, and as we saw from the proof of Lemma 2, in this case the IRF never crosses the steady state. In fact, from the proof of Lemma 1 we also have the stronger result that $\hat{b}_{ht} = 0$, and hence the IRF is

$$a_{bk} = 0 \text{ for } k = 0, 1, 2, 3, \ldots$$

□

C.4 PROPOSITION 1:

PROPOSITION 1 (UIP Violations). *The magnitude and direction of the UIP regression coefficients $\beta_k = \frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*}$ depend on the monetary-fiscal policy mix as follows.*

(i) ***Active Monetary, Passive Fiscal policy*** $(\phi_{\pi 1} > 1, \kappa_b \in (\theta - \theta_2, \frac{\theta+(\theta_2-1)\rho_\tau}{1-\rho_\tau})$***:***

(a) $\rho_\tau \leq \underline{\rho}(\kappa_b)$: UIP violations conform with the classic UIP puzzle at all horizons and decline monotonically to zero:

$$\beta_k < 0 \textit{ for } k = 1, 2, 3, \ldots$$

(b) $\rho_\tau > \underline{\rho}(\kappa_b)$: UIP violations exhibit cyclical (cosine) dynamics, being negative at

short horizons, but eventually positive, for at least some periods:

$$\beta_k < 0 \text{ for } k < \bar{k}$$
$$\beta_k > 0 \text{ for some } k > \bar{k}$$

where $\bar{k} > 1$*.*

(ii) ***Passive Monetary, Active Fiscal policy*** $(\phi_\pi < 1, \kappa_b \in (0, \theta - \theta_2)$***:*** *UIP violations go in the same direction at all horizons and are in fact always zero:*

$$\beta_k = 0 \text{ for } k = 1, 2, 3, \dots$$

Proof. **Part (i), sub-point (a):** Start with the definition of the UIP regression coefficient,

$$\beta_k = \frac{\text{Cov}(\hat{\lambda}_{t+k}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)}$$

and note that in equilibrium the expected excess returns are linear in bond holdings,

$$E_t(\hat{\lambda}_{t+1}) = -\chi_b \hat{b}_{ht} \tag{C.4}$$

where $\chi_b = -\frac{b_h}{\beta(1+i)u_c}\left((u_{b_h b_h} - \frac{u_{b_h} u_{c b_h}}{u_c}) - (u_{b_h b_f} - \frac{u_{b_f} u_{c b_h}}{u_c})\right) s$. Where $u_x(.)$ and $u_{xy}(.)$ respectively are the steady state values of the first and second partial derivative of the utility function. In the symmetric steady state, $u_{b_f} = u_{b_h}$ and given the assumption of imperfect substitutability between home and foreign bonds (and since utility is concave):

$$u_{b_h b_h} < u_{b_f b_f} < 0$$

it follows that $\chi_b > 0$. By Lemma 2, we know that in this case (Active Monetary policy), the IRF of $\hat{b}_{ht}$ is positive at all horizons (i.e. $a_{bk} > 0$ for all k), and next, I will show that the IRF of the interest rate differential $\hat{i}_t - \hat{i}_t^*$ is also always positive. Then by (C.4) we can conclude that $\beta_k < 0$ for all $k \geq 1$.

To derive the IRF of the interest rate differential, note that since the foreign interest rate is constant, $\hat{i}_t - \hat{i}_t^* = \hat{i}_t = \phi_\pi + v_t$. From Lemma 1, the equilibrium inflation is given by

$$\hat{\pi}_t = \gamma_b^\pi \hat{b}_{ht} + \gamma_\tau^\pi \hat{\tau}_t - \frac{v_t}{\phi_\pi}$$

where $\gamma_b^\pi = \gamma_M + \frac{\theta_2(\phi_\pi - \rho_\tau)(\gamma_\Psi - \gamma_M(\phi_\pi - 1))}{\phi_\pi(\kappa_b(1-\rho_\tau) + \theta_2(\phi_\pi - \rho_\tau)) - \theta(\phi_\pi - \rho_\tau)} > 0$, $\gamma_\tau^\pi = -\frac{\tau}{b_h}\frac{\rho_\tau(\gamma_\Psi - \gamma_M(\phi_\pi - 1))}{\phi_\pi(\kappa_b(1-\rho_\tau) + \theta_2(\phi_\pi - \rho_\tau)) - \theta(\phi_\pi - \rho_\tau)}$. Thus,

$$\begin{aligned}
\hat{i}_t - \hat{i}_t^* &= \phi_\pi(\gamma_b^\pi \hat{b}_{ht} + \gamma_\tau^\pi \hat{\tau}_t) \\
&= \phi_\pi((\gamma_b^\pi a_{b0} + \gamma_\tau^\pi a_{\tau 0})v_t + (\gamma_b^\pi a_{b1} + \gamma_\tau^\pi a_{\tau 1})v_{t-1} + \dots) \\
&= a_{i0} v_t + a_{i1} v_{t-1} + a_{i2} v_{t-2} + \dots
\end{aligned}$$

where I have substituted in the Wold decomposition of $\hat{b}_{ht}$ and $\hat{\tau}_t$, and by the proof of Lemma 2, $a_{bk} = \frac{1+i}{\phi_\pi} a_{11}^{(k)}$ and $a_{\tau k} = \frac{1+i}{\phi_\pi} a_{21}^{(k)}$, with $a_{lm}^{(k)}$ the (k, l) element of the matrix A^k. This defines the Wold decomposition of the interest rate differential through the coefficients a_{ik}, where

$$a_{ik} = \phi_\pi(\gamma_b^\pi a_{bk} + \gamma_\tau^\pi a_{\tau k}) = (1+i)\left(\gamma_b^\pi \frac{\lambda_1^k p_{11} - \lambda_2^k p_{22}}{|P|} + \gamma_\tau^\pi \frac{\lambda_1^k - \lambda_2^k}{|P|}\right)$$
$$= (1+i)\left(\frac{\lambda_1^k}{|P|}(p_{11}\gamma_b^\pi + \gamma_\tau^\pi) - \frac{\lambda_2^k}{|P|}(p_{12}\gamma_b^\pi + \gamma_\tau^\pi)\right)$$

and $\lambda_1 > \lambda_2 > 0$ are the ordered eigenvalues of A, and P is the matrix of eigenvectors, with $p_{11} = \frac{\lambda_1 - \rho_\tau}{(1-\rho_\tau)\kappa_b}\frac{b_h}{\tau}$, and $p_{12} = \frac{\lambda_2 - \rho_\tau}{(1-\rho_\tau)\kappa_b}\frac{b_h}{\tau}$. Since the eigenvalues are ordered and positive, $p_{11} > p_{12} > 0$, and hence $|p_{11}\gamma_b^\pi + \gamma_\tau^\pi| > |p_{12}\gamma_b^\pi + \gamma_\tau^\pi|$. If $\gamma_\tau^\pi > 0$ then it follows that $p_{11}\gamma_b^\pi + \gamma_\tau^\pi > 0$, and thus $\left(\frac{\lambda_1^k}{|P|}(p_{11}\gamma_b^\pi + \gamma_\tau^\pi) - \frac{\lambda_2^k}{|P|}(p_{12}\gamma_b^\pi + \gamma_\tau^\pi)\right) > 0$ and hence $a_{ik} > 0$.

On the other hand, if $\gamma_\tau^\pi < 0$, first we need to show $p_{11}\gamma_b^\pi + \gamma_\tau^\pi > 0$. Start with,

$$\begin{aligned} p_{11}\gamma_b^\pi - |\gamma_\tau^\pi| &\propto (\theta - \kappa_b(1-\rho_\tau) - \theta_2\rho_\tau + \sqrt{(\theta - \kappa_b(1-\rho_\tau) + \theta_2\rho_\tau)^2 - 4\theta\theta_2\rho_\tau})(\gamma_\Psi(\phi_\pi - \rho_\tau) - \gamma_M(i(\phi_\pi - \rho_\tau) - \kappa_b(1-\rho_\tau)\phi_\pi)) \\ &\quad - 2\kappa_b\theta_2(1-\rho_\tau)\rho_\tau(\gamma_\Psi + \gamma_M(1-\phi_\pi)) \\ &\geq (\theta - \kappa_b(1-\rho_\tau) - \theta_2\rho_\tau)(\gamma_\Psi(\phi_\pi - \rho_\tau) - \gamma_M(i(\phi_\pi - \rho_\tau) - \kappa_b(1-\rho_\tau)\phi_\pi)) - 2\kappa_b\theta_2(1-\rho_\tau)\rho_\tau(\gamma_\Psi + \gamma_M(1-\phi_\pi)) \end{aligned}$$

The last equation is concave and quadratic in κ_b, so if it is positive for any $k_1 < k_2$, then it's positive for all values in between as well. Furthermore, note that in order for the the eigenvalues to be real and less than one in magnitude we must have $\kappa_b \in (\theta - \theta_2, \frac{\theta + \theta_2\rho_\tau - 2\sqrt{\theta\theta_2\rho_\tau}}{1-\rho_\tau}]$, and thus it is enough to show that the quadratic equation is positive at both ends of this interval.

For $\kappa_b = \theta - \theta_2$,

$$p_{11}\gamma_b^\pi - |\gamma_\tau^\pi| \geq \gamma_\Psi(1-\rho_\tau)(\theta_2 + \theta\rho_\tau - 2\theta_2(1+\theta-\theta_2)\rho_\tau) + \gamma_M((\theta - \theta_2 - i)(1-\rho_\tau)(\theta_2 + \theta\rho_\tau - 2\theta_2\rho_\tau)$$

and since $\rho_\tau \in [0, \frac{\theta_2}{\theta})$ it follows that $(\theta_2 + \theta\rho_\tau - 2\theta_2\rho_\tau) > 0$, and $(\theta_2 + \theta\rho_\tau - 2\theta_2(1+\theta-\theta_2)\rho_\tau) > 0$. Also $\theta - \theta_2 - i = (1+i)\gamma_\Psi > 0$, and hence $p_{11}\gamma_b^\pi - |\gamma_\tau^\pi| > 0$.

On the other hand, if $\kappa_b = \frac{\theta + \theta_2\rho_\tau - 2\sqrt{\theta\theta_2\rho_\tau}}{1-\rho_\tau}$:

$$\begin{aligned} p_{11}\gamma_b^\pi - |\gamma_\tau^\pi| &\geq 2\gamma_\Psi((1-\rho_\tau)\sqrt{\theta\theta_2\rho_\tau} - \theta_2\rho_\tau(\theta_2\rho_\tau + 1 + \theta - \rho_\tau - 2\sqrt{\theta\theta_2\rho_\tau}) + 2\gamma_M(\sqrt{\theta\theta_2\rho_\tau} - \theta_2\rho_\tau)(\theta + \theta_2\rho_\tau - 2\sqrt{\theta\theta_2\rho_\tau} - (1-\rho_\tau)i) \\ &= 2\gamma_\Psi((1-\rho_\tau)(\sqrt{\theta\theta_2\rho_\tau} - \theta_2\rho_\tau) - \theta_2\rho_\tau(\theta - 2\sqrt{\theta\theta_2\rho_\tau} + \theta_2\rho_\tau)) + 2\gamma_M\sqrt{\theta_2\rho_\tau}(\sqrt{\theta} - \sqrt{\theta_2\rho_\tau})(\theta + \theta_2\rho_\tau - 2\sqrt{\theta\theta_2\rho_\tau} - (1-\rho_\tau)i) \\ &= 2\sqrt{\theta_2\rho_\tau}(\sqrt{\theta} - \sqrt{\theta_2\rho_\tau})\left(\gamma_\Psi((1-\rho_\tau) - \sqrt{\theta_2\rho_\tau}(\sqrt{\theta} - \sqrt{\theta_2\rho_\tau})) + \gamma_M((\sqrt{\theta} - \sqrt{\theta_2\rho_\tau})^2 - i(1-\rho_\tau))\right) \\ &= 2\sqrt{\theta_2\rho_\tau}(\sqrt{\theta} - \sqrt{\theta_2\rho_\tau})\underbrace{\left((\gamma_\Psi - \gamma_M i)(1-\rho_\tau) + \gamma_\Psi\theta_2\rho_\tau + \gamma_M(\theta + \theta_2\rho_\tau) - (2\gamma_M + \gamma_\Psi)\sqrt{\theta\theta_2\rho_\tau}\right)}_{=\Omega} \end{aligned}$$

Since $\rho_\tau < \frac{\theta_2}{\theta}$ and $\theta_2 < \theta_1$ it follows that $\sqrt{\theta} > \sqrt{\theta_2\rho_\tau}$. To evaluate the second piece in parenthesis (which I have named Ω for brevity), substitute in $\theta = (1+i)(1+\gamma_\Psi+\gamma_M)$ and $\theta_2 = 1 + \gamma_M(1+i)$ and simplify to get:

$$\Omega = ((\gamma_\Psi + \gamma_M)(1 + \gamma_M(1+i)) + \gamma_M(1+\gamma_\Psi+\gamma_M)(1+i)\rho_\tau)^2 - (1+\gamma_\Psi+\gamma_M)(\gamma_\Psi+2\gamma_M)^2(1+i)(1+\gamma_M(1+i))\rho_\tau$$

This is a convex quadratic equation in ρ_τ, with zeros at $\rho_1 = \frac{\theta_2}{\theta}$ and at $\rho_2 = \frac{\theta_2}{\theta}\frac{(\gamma_\Psi+\gamma_M)^2}{\gamma_M^2}$, and since $\gamma_\Psi > 0$, $\rho_1 < \rho_2$. Therefore, $\Omega > 0$ for all $\rho_\tau < \frac{\theta_2}{\theta}$ and thus we conclude that $p_{11}\gamma_b^\pi - |\gamma_\tau^\pi| > 0$.

Thus, we have shown that under Active Monetary Policy we have $p_{11}\gamma_b^\pi - |\gamma_\tau^\pi| > 0$, and thus since $\lambda_1 > \lambda_2 > 0$ we have $\left(\frac{\lambda_1^k}{|P|}(p_{11}\gamma_b^\pi + \gamma_\tau^\pi) - \frac{\lambda_2^k}{|P|}(p_{12}\gamma_b^\pi + \gamma_\tau^\pi)\right) > 0$. Therefore, under Active Monetary policy, $a_{ik} > 0$ for all k.

Plugging this and the IRF for $\hat{b}_{ht}$ in the UIP regression coefficients, I obtain

$$\begin{aligned}\beta_k &= -\chi_b \frac{\text{Cov}(\hat{b}_{h,t+k-1}, \hat{i}_t - \hat{i}_t^*)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} \\ &= -\chi_b \frac{\sigma_v^2(a_{b,k-1}a_{i,0} + a_{b,k}a_{i,1} + a_{b,k+1}a_{i,2} + \dots)}{\text{Var}(\hat{i}_t - \hat{i}_t^*)} \\ &< 0\end{aligned}$$

where the inequality follows from $\chi_b > 0$ and $a_{bk} > 0$ and $a_{ik} > 0$ for all k.

Above we implicitly assumed that A is diagonalizable. But the proof is very similar if it is not, with the only difference being that

$$\begin{aligned}a_{ik} = \phi_\pi(\gamma_b^\pi a_{bk} + \gamma_\tau^\pi a_{\tau k}) &= (1+i)\left(\gamma_b^\pi \lambda^{k-1}(\lambda + k\frac{p_{11}}{p_{12}-p_{11}}) + \gamma_\tau^\pi k \frac{\lambda^{k-1}}{p_{12}-p_{11}}\right) \\ &= (1+i)\left(\gamma_b^\pi \lambda^k + \frac{k\lambda^{k-1}}{p_{12}-p_{11}}(\gamma_b^\pi p_{11} + \gamma_\tau^\pi)\right)\end{aligned}$$

But we have already shown $(\gamma_b^\pi p_{11} + \gamma_\tau^\pi) > 0$, and by the proof of Lemma 2, $p_{12} - p_{11} > 0$, hence $a_{ik} > 0$ for all k again, and we are done.

Part (i), sub-point (b): Here I work under the assumption that the roots are complex - i.e. $\rho_\tau > \underline{\rho}(\kappa_b)$ as defined in Lemma 2. We can express the UIP regression coefficients as

$$\beta_k = \frac{\text{Cov}(-\chi_b E_t(\hat{b}_{h,t+k-1}), \phi_\pi(\gamma_b^\pi \hat{b}_{ht} + \gamma_\tau^\pi \hat{\tau}_t))}{\text{Var}(\phi_\pi(\gamma_b^\pi \hat{b}_{ht} + \gamma_\tau^\pi \hat{\tau}_t))} = -\chi_b\phi_\pi(\gamma_b^\pi \frac{\text{Cov}(E_t(\hat{b}_{h,t+k-1}), \hat{b}_{ht})}{\text{Var}(\phi_\pi(\gamma_b^\pi \hat{b}_{ht} + \gamma_\tau^\pi \hat{\tau}_t))} + \gamma_\tau^\pi \frac{\text{Cov}(E_t(\hat{b}_{h,t+k-1}), \hat{\tau}_{ht})}{\text{Var}(\phi_\pi(\gamma_b^\pi \hat{b}_{ht} + \gamma_\tau^\pi \hat{\tau}_t))})$$

Since $E_t(\hat{b}_{t+k}) = [1, 0]A^k x_t$, we have

$$\text{Cov}(E_t(\hat{b}_{t+k}), b_t) = a_{11}^{(k)} \text{Var}(\hat{b}_t) + a_{12}^{(k)} \text{Cov}(\hat{b}_t, \hat{\tau}_t) \tag{C.5}$$

$$\text{Cov}(E_t(\hat{b}_{t+k}), \tau_t) = a_{11}^{(k)} \text{Cov}(\hat{b}_t, \hat{\tau}_t) + a_{12}^{(k)} \text{Var}(\hat{\tau}_t) \tag{C.6}$$

Compute the variance on both sides of the tax policy rule to obtain

$$Var(\hat{\tau}_t) = \frac{b_h^2}{\tau^2}\frac{k_b^2}{(1+\rho_\tau)}(1-\rho_\tau)Var(\hat{b}_t) + 2\frac{b_h}{\tau}\frac{\kappa_b\rho_\tau}{(1+\rho_\tau)}Cov(\hat{\tau}_t, b_t)$$

and then combine with

$$\operatorname{Cov}(\hat{\tau}_t,\hat{b}_t) = \operatorname{Cov}(\rho\hat{\tau}_{t-1} + a_{21}^{(1)}\hat{b}_{t-1}, a_{11}^{(1)}\hat{b}_{t-1} + a_{12}^{(1)}\hat{\tau}_{t-1} + \frac{1+i}{\phi_\pi}v_t)$$

$$= -\frac{\rho_\tau^2}{\theta_2}\frac{\tau}{b}\operatorname{Var}(\hat{\tau}_t) + \frac{\theta-(1-\rho_\tau)\kappa_b}{\theta_2}(1-\rho_\tau)\kappa_b\frac{b}{\tau}\operatorname{Var}(\hat{b}_t) + (\frac{\theta-(1-\rho_\tau)\kappa_b}{\theta_2}\rho_\tau - (1-\rho_\tau)\kappa_b\frac{\rho_\tau}{\theta_2})\operatorname{Cov}(\hat{b}_t,\hat{\tau}_t)$$

to obtain

$$\operatorname{Cov}(\hat{\tau}_t,\hat{b}_t) = \underbrace{(1-\rho_\tau)\kappa_b\frac{b_h}{\tau}\frac{(\theta(1+\rho_\tau)-\kappa_b)}{\theta_2+\rho_\tau(\theta_2+2\kappa_b-\theta(1+\rho_\tau))}}_{=\delta}\operatorname{Var}(\hat{b}_t).$$

Substituting this back in (C.5) yields $\operatorname{Cov}(E_t(\hat{b}_{t+k}),b_t) = (a_{11}^{(k)} + \delta a_{12}^{(k)})\operatorname{Var}(\hat{b}_{ht})$, and similarly substituting things out in (C.6) yields $\operatorname{Cov}(E_t(\hat{b}_{t+k}),\hat{\tau}_t) = (a_{11}^{(k)}\delta + a_{12}^{(k)}((\frac{b_h}{\tau})^2\frac{\kappa_b^2(1-\rho_\tau)}{1+\rho_\tau} + 2\frac{b_h}{\tau}\frac{\kappa_b\rho_\tau\delta}{1+\rho_\tau}))\operatorname{Var}(\hat{b}_{ht})$, and hence the UIP coefficient becomes

$$\beta_{k+1} = -\frac{\chi_b\phi_\pi\operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}(\gamma_b^\pi(a_{11}^{(k)}+\delta a_{12}^{(k)}) + \gamma_\tau^\pi(a_{11}^{(k)}\delta + a_{12}^{(k)}((\frac{b_h}{\tau})^2\frac{\kappa_b^2(1-\rho_\tau)}{1+\rho_\tau} + 2\frac{b_h}{\tau}\frac{\kappa_b\rho_\tau\delta}{1+\rho_\tau})))$$

$$= -\frac{\chi_b\phi_\pi\operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}\left(a_{11}^{(k)}\underbrace{(\gamma_b^\pi+\gamma_\tau^\pi\delta)}_{=\gamma_{a_{11}}} + a_{12}^{(k)}\underbrace{(\gamma_b^\pi\delta + \gamma_\tau^\pi(((\frac{b_h}{\tau})^2\frac{\kappa_b^2(1-\rho_\tau)}{1+\rho_\tau} + 2\frac{b_h}{\tau}\frac{\kappa_b\rho_\tau\delta}{1+\rho_\tau})))}_{=\gamma_{a_{12}}}\right)$$

$$= -\frac{\chi_b\phi_\pi\operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}\left(a_{11}^{(k)}\gamma_{a_{11}} + a_{12}^{(k)}\gamma_{a_{12}}\right)$$

$$= -\frac{\chi_b\phi_\pi\operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}\left(\gamma_{a_{11}}|\lambda|^k(\cos(k\zeta)+\frac{x}{y}\sin(k\zeta)) - \gamma_{a_{12}}|\lambda|^k\frac{x^2+y^2}{y^2}\sin(k\zeta)\right)$$

$$= -\frac{\chi_b\phi_\pi\operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}|\lambda|^k\left(\gamma_{a_{11}}\cos(k\zeta) + (\gamma_{a_{11}}\frac{x}{y} - \gamma_{a_{12}}\frac{x^2+y^2}{y^2})\sin(k\zeta))\right)$$

$$= -\frac{\chi_b\phi_\pi\operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}|\lambda|^k\underbrace{\sqrt{\gamma_{a_{11}}^2 + (\gamma_{a_{11}}\frac{x}{y} - \gamma_{a_{12}}\frac{x^2+y^2}{y^2})^2}}_{=\Gamma}\cos(k\zeta+\psi-\frac{\pi}{2})$$

$$= -\frac{\chi_b\phi_\pi\operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}|\lambda|^k\Gamma\cos(k\zeta+\psi-\frac{\pi}{2})$$

where $\psi = \arctan(\frac{\gamma_{a_{11}}}{\gamma_{a_{11}}\frac{x}{y}-\gamma_{a_{12}}\frac{x^2+y^2}{y^2}}) + \pi\mathbb{I}(\frac{\gamma_{a_{11}}}{\gamma_{a_{11}}\frac{x}{y}-\gamma_{a_{12}}\frac{x^2+y^2}{y^2}} < 0)$, and x and y are the real and imaginary part of the eigenvectors as defined in Lemma 2, and $\zeta = \arctan(\frac{b}{a}) \in [0,\frac{\pi}{2})$ where the eigenvalue is $a+bi$. I am also using the convention that $a_{11}^{(0)} = 1$ and $a_{12}^{(0)} = 0$.

This gives us the general expression of β_k and shows that it is cyclical, and changes sign as the cosine expression changes sign. Lastly, I will show that $\beta_1 < 0$, which finishes the proof by establishing that the regression coefficients start negative, and then will eventually turn positive as k grows (since $\zeta \in [0, \frac{\pi}{2})$).

To show $\beta_1 < 0$, start by re-writing it as $\beta_1 = -\frac{\chi_b \phi_\pi \operatorname{Var}(\hat{b}_{ht})}{Var(\hat{i}_t)}(\gamma_b^\pi + \gamma_\tau^\pi \delta)$ by using the fact that $a_{11}^{(0)} = 1$ and $a_{12}^{(0)} = 0$, and notice that it is enough to show that $\gamma_b^\pi + \delta\gamma_\tau^\pi > 0$. Substitute in the definitions for the three variables, bring everything to a common denominator, and since the resulting denominator is positive, the sign of $\gamma_b^\pi + \delta\gamma_\tau^\pi$ is the same as the sign of the numerator:

$$(\theta_2(1+\rho_\tau) + \rho_\tau(2\kappa_b - \theta(1+\rho_\tau))(\gamma_\Psi(\phi_\pi - \rho_\tau) + \gamma_M\kappa_b(1-\rho_\tau) - \gamma_M(\phi_\pi - \rho_\tau)) - \kappa_b(1-\rho_\tau)\rho_\tau(\theta(1+\rho_\tau - \kappa_b)(\gamma_\Psi - \gamma_M(\phi_\pi - 1)) \tag{C.7}$$

This is a convex quadratic function of κ_b ($\frac{\partial^2 .}{(\partial\kappa_b)^2} = 2(1-\rho_\tau)\rho_\tau(\gamma_\Psi + \gamma_M(\phi_\pi + 1)) > 0$), and I will show that it is positive for all $\kappa_b > \theta - \theta_2$, by showing that it is positive and increasing at $\kappa_b = \theta - \theta_2$.

At $\kappa_b = \theta - \theta_2$, the expression becomes

$$\gamma_\Psi(1-\rho_\tau)(\theta_2 + \theta\rho_\tau)(\phi_\pi\theta_2 - \rho_\tau\theta) > \gamma_\Psi(1-\rho_\tau)(\theta_2 + \theta\rho_\tau)(\phi_\pi\theta_2 - \theta_2) > 0$$

where the first inequality follows from $\rho_\tau < \frac{\theta_2}{\theta}$, and the second from $\phi_\pi > 1$.

On the other hand, its derivative at $\kappa_b = \theta - \theta_2$ is:

$$\begin{aligned}\gamma_\Psi\rho_\tau(\theta(1-\rho_\tau)^2 + 2(\phi_\pi - \rho_\tau - \theta_2(1-\rho_\tau))) + \gamma_M(\theta(1-\rho_\tau)\rho_\tau(2\phi_\pi + 1 - \rho_\tau) + 2i\rho_\tau(\phi_\pi - \rho_\tau) + \theta_2(1-\rho_\tau)(\phi_\pi(1-\rho_\tau) - 2\rho_\tau)) \\ > (1-\rho_\tau)^2(\gamma_M\theta_2 + (\gamma_\Psi + \gamma_M)\theta\rho_\tau) \\ > 0\end{aligned}$$

where the first inequality follows from the fact that the top line is increasing in ϕ_π and $\phi_\pi > 1$. Thus, we have shown that (C.7) is positive and increasing at $\kappa_b = \theta - \theta_2$, and hence $\gamma_b^\pi + \delta\gamma_\tau^\pi > 0$ which implies that $\beta_1 > 0$. This completes the proof of part (i), sub-point b.

Part (ii): By the proof of Lemma 3 the eigenvalues of A are always real in this case, and by similar steps to the proof of Proposition 1, Part (i), sub-point (a) we can show that the IRF of $\hat{i}_t$ is positive at all horizons and hence β_k has the same sign for all k. Moreover, from Lemma 3 we have the particular result that $\hat{b}_{h,t+k} = 0$ for all k, and hence

$$\beta_k = -\chi_b \frac{\operatorname{Cov}(\hat{b}_{h,t+k-1}, \hat{i}_t - \hat{i}_t^*)}{\operatorname{Var}(\hat{i}_t - \hat{i}_t^*)} = -\chi_b \frac{\operatorname{Cov}(0, \hat{i}_t - \hat{i}_t^*)}{\operatorname{Var}(\hat{i}_t - \hat{i}_t^*)} = 0$$

□

D Model Discussion

D.1 Forward Exchange Rate Contracts and UIP Violations

In this section, I augment the model to include trade in forward contracts on currencies, and show that trading in forward contracts creates a synthetic position long one country's bond and short the other. Hence, it does not matter whether one implements carry trades through forward contracts, or through trades in the bonds themselves, as both trading strategies earn the same convenience yield differential, which violates UIP. In other words, the convenience yield mechanism generates UIP violations that emerge both when looking at exchange rates and interest rates data only, and when only looking at forward and spot exchange rates.

The key to the result is that in a model with bond convenience yields, the Covered Interest Parity (CIP) holds if and only if a covered position in foreign currency is equivalent to a position in home bonds, both in financial terms and in convenience benefits. Why is that? Notice that a covered position in EUR risk-free bonds, where the future interest rate $(1 + i_t^*)$ has been sold forward for dollars at the equilibrium rate F_t, generates a risk-free USD payoff, and not a risk-free EUR payoff. As such, it has a comparable convenience value to the other risk-free USD asset - US Treasuries. Being a risk-free USD asset, it carries the convenience benefits of USD risk-free assets, because it allows the investor to pledge a sure, future amount of USD, and not a sure future amount of EUR. Another way to think about it, is that a covered position in EUR bonds is in fact long USD, and not long EUR. Similarly, buying foreign currency forward is a strategy long in foreign currency and short home currency. It simultaneously increases the pledgeable amount of foreign currency proceeds and decreases the pledgeable amount of home currency, hence it creates a synthetic, zero-cost position that is long home bonds and short foreign bonds, and thus in equilibrium, on average it earns the convenience yield differential.

The CIP condition states that investing in foreign risk-free bonds and using forward contracts to eliminate the exchange rate risk must yield the same rate of return as investing in home bonds. CIP has been shown to hold in the data very well, outside of a few, short-lived episodes during times of extraordinary financial markets turbulence (e.g. some days during the recent financial crisis). The intuition behind the condition is that the covered position in foreign bonds is a risk-free asset denominated in home currency, and not in foreign currency, hence in equilibrium it must have the same rate of return as the domestic bond (another domestic risk-free asset), or otherwise there will be an arbitrage opportunity.

To be more concrete, let F_t denote the equilibrium USD-EUR forward rate, so that today we can agree to trade 1 EUR tomorrow in exchange of F_t USD. Imagine then that an investor borrows \$1 today at the interest rate $1 + i_t$, changes it into $\frac{1}{S_t}$ EURs and invests it at the interest rate $1+i_t^*$, and at the same time has sold forward the proceeds at the forward rate F_t. Thus, his payoff from the covered foreign position is $\frac{F_t}{S_t}(1 + i_t^*)$ and the cost of the 1 USD is $1 + i_t$ and CIP states:

$$1 + i_t = \frac{F_t}{S_t}(1 + i_t^*),$$

so that a position in a US Treasury has an equivalent financial return to a covered position in EUR denominated government bonds (e.g. German Bunds). A position in US Treasuries also carries the convenience benefit $\Psi_{b_h,t}$ and the covered position in foreign bonds is another risk-free USD asset which carries the (possibly different) convenience benefit $\tilde{\Psi}_{b_h,t}$. Conditional on CIP, the convenience benefits of the two positions must be the same:

$$\Psi_{b_h,t} = \tilde{\Psi}_{b_h,t}.$$

This follows from the fact that an investment in US Treasuries carries a total return of $1+i_t+\Psi_{b_h,t}$, the sum of the financial return and the convenience benefit, and an investment in a covered position in EUR denominated bonds similarly carries a total return of $\frac{F_t}{S_t}(1+i_t^*)+$

$\tilde{\Psi}_{b_h,t}$. The two risk-free returns must be equal, otherwise there is an arbitrate opportunity. Given that CIP restricts the financial returns to be equal, it follows that the convenience benefits must be equal as well: $\Psi_{b_h,t} = \tilde{\Psi}_{b_h,t}$.

Thus, when CIP holds (as it does in the data) and bonds offer convenience benefits (as is also true in the data), in equilibrium, covered position in foreign bonds, which yield a risk-free payoff in the home currency and not a payoff in foreign currency, must offer the same convenience benefits as an equivalent position in home currency bonds.

This leads to the important result that (in log-approximation) the expected return on buying foreign currency forward (a popular way of implementing the carry trade without the need to transact in bond markets) is:

$$E_t(s_{t+1} - f_t) = E_t(\Delta s_{t+1} + i_t^* - i_t) = \hat{\Psi}_{b_h,t} - \hat{\Psi}_{b_f,t}.$$

This shows that taking positions in the forwards market is akin to creating a synthetic position that is simultaneously long foreign currency bonds and short home currency bonds. This is of course, very intuitive, as buying foreign currency forward is a contract long in the foreign currency and short in the home currency. Entering into this contract reduces the amount of future USD the investor is able to pledge today as collateral (since he has already sold this USD for EURs) and at the same time increase the pledgeable amount of EUR. At the end of the day, the strategy implemented through forwards market has equivalent financial and convenience returns to a trade in the home and foreign bonds themselves, hence the forwards data would display equivalent UIP violations and the mechanism works in the same way. Due to this equivalence and for simplicity, the benchmark model abstracts from trade in forward contracts.

D.2 Interest Rates Across Different Types of Assets

It seems reasonable to think that some assets, like Treasuries, tend to have bigger convenience yields than other short-term assets, like say inter-bank loans. Does the model then imply that the interest rate differential (across countries) on Treasuries would behave very differently than the interest rate differentials of other, less liquid assets? That would be a potential concern, because in the data interest rate differentials across countries behave very similarly, no matter what type of short-term rate one uses.

Re-assuringly, the model has no such counter-factual implications. In the model, the primary difference between different types of interests rates is in their *level*, where the interest rate of an asset with a lower convenience yield is generally higher, but the overall dynamics of interest rates across different types of assets is remarkably similar. In particular, the interest rate of a hypothetical asset that has *no* convenience yield, call it $\tilde{i}_t$, has almost identical dynamics, and is highly correlated with the interest rate of the Treasury bill, i_t. As a result, the interest rate differentials across different types of assets are also quite similar.

For example, in the benchmark calibration the correlation between the two interest rates is 0.78, and their time series properties are very similar – the autocorrelation of the T-bill interest rate is 0.866 and that of $\tilde{i}_t$ is 0.843. Moreover, the standard deviation of $\tilde{i}_t$ is 0.0032 and that of i_t is 0.004. And this is just a conservative lower bound on the similarity we could expect to see in the data, since there we observe assets that have lower, but still positive convenience yields (i.e. Commercial Paper). A hypothetical asset with *some* convenience yield, will look even more akin to the Treasury's in the model.

The reason for this similarity is the fundamentally negative correlation between the convenience yield and the Treasury interest rate – when the convenience yield is high, then the interest rate on the Treasury is low as investors require a lower financial compensation to hold that asset (the correlation is -0.63 in the benchmark calibration). However, this countervailing force helps make $\tilde{i}_t$ behave similarly to i_t. To see this clearly, note that the equilibrium condition linking the two interest rates in the model is

$$\tilde{i}_t = i_t + \hat{\Psi}_t^H$$

As we saw in the main text, contractionary shocks increase i_t while lowering $\hat{\Psi}_t^H$ – this is the key feature generating the UIP Puzzle, since it leads to the result that high interest rates are associated with high excess currency returns (which compensate for the low $\hat{\Psi}_t^H$). However, this exact same mechanism also leads to an increase in $\tilde{i}_t$, which generates a positive correlation between i_t and $\tilde{i}_t$. Lastly, the convenience yield is considerably less volatile than the Treasury interest rate itself – the std deviation of $\hat{\Psi}_t^H$ is only half of that of i_t. These forces together result in a high, positive correlation between i_t and $\tilde{i}_t$.

Thus, the bottom line is that the model implies that the interest rates on different types of assets, some more liquid than others, will be highly correlated and overall behave very similarly. Just like what we observe in the data.

D.3 Long-term Bonds

It is well known that the UIP holds better in the "long-run". Specifically, Chinn (2006) and others have shown that 5-year (and longer) excess currency returns display smaller UIP deviations, than the typical estimates of the UIP Puzzle in short-term bonds. It is important to note that the model can match this observation, even if we make the strong (and counterfactual) assumption that long-term bonds are perfect substitutes for short-term bonds in terms of liquidity, and hence earn the *same* convenience yield.

The key empirical result centers on the regression

$$s_{t+N} - s_t + R_t^{*,(N)} - R_t^{(N)} = \alpha^{(N)} + \beta^{(N)}(R_t^{(N)} - R_t^{*,(N)}) + \varepsilon_{t+N}^{(N)}$$

where the $R_t^{(N)} = N * i_t^{(N)}$ is the cumulative interest rate on a $N-$period bond. The left-hand variable is the excess return on N-period foreign bond over a N-period home bond when both are held to maturity. It turns out, that while $\beta^{(N)}$ is large and significantly negative for $N \leq 1$ years, the estimates are smaller and often insignificant for $N \geq 5$ years. In other words, long-term bond returns appear to be equalized across countries, even though the short-term bonds display a clear violation of UIP.

In the model, this observation is trivially true if we assume that long-term bonds do not offer any of the convenience benefits of short-term bonds. But the point of this section is to show that the relation will still hold, *even if* long-term bonds are perfect substitutes for short-term bonds. The intuition is that multi-period excess currency returns offset the sum of expected convenience yield differentials that accrue throughout the life of the bond. So if we are looking at a 5-year bond, then the 5-year cumulative excess return will equal the expected sum of convenience yield differentials for the next 5 years. Crucially, the convenience yield differential switches signs at longer horizons (recall that this is what generates the reversal in UIP violations), and thus for long-term bonds (in particular 7+ years) the sum of expected convenience yield differentials is roughly zero. Thus, long-term excess currency returns end up being equalized, even though the short-term excess returns are not, due to the cyclical movements in the convenience yield differential analyzed in the main body of the text.

To make this concrete, assume that the convenience benefit is again derived from a similar transaction cost function $\Psi(c_t, m_t, b_t^T, b_t^{*,T})$, where b_t^T this time is the total amount of home bonds in the agent's portfolio:

$$b_t^T = b_t^{(2)} + b_t^{(2)} + \dots$$

and $b_t^{*,T}$ is similarly the total amount of foreign bonds owned. Thus, the short-term bonds are no longer special relative to the longer maturity ones – they all enter equivalently in the transaction costs function.

The resulting Euler equation for 1-period bonds is the same as before:

$$E_t(\Delta s_{t+1} + i_t^* - i_t) = \hat{\Psi}_t^H - \hat{\Psi}_t^F \tag{D.1}$$

where $\hat{\Psi}_t^H$ and $\hat{\Psi}_t^F$ are the log-linearized home and foreign convenience yields. Note that the convenience yields on bonds across all maturities are the same, because the derivatives of the transaction cost $\Psi(.)$ in terms of different maturities are equal. That is, all bonds of the same currency denomination are equivalent to each other in terms of liquidity.

We can derive a similar Euler equation for an arbitrary N-periods to maturity bond:

$$E_t(\Delta s_{t+1} + \hat{p}_{t+1}^{*,(N-1)} - \hat{p}_t^{*,(N)} - (\hat{p}_{t+1}^{(N-1)} - \hat{p}_t^{(N)}) = \hat{\Psi}_t - \hat{\Psi}_t^*$$

where $\hat{p}_t^{(N)}$ is the (log-linearized) price of the N period (zero-coupon) bond. The cumulative interest rate payments of the bond are $R_t^{(N)} = N * i_t^{(N)} = \frac{1}{p_t^N}$, and hence

$$E_t(\Delta s_{t+1} + \hat{R}_{t+1}^{*,(N)} - \hat{R}_{t+1}^{(N)} - (\hat{R}_t^{*,(N-1)} - \hat{R}_t^{(N-1)})) = \hat{\Psi}_t - \hat{\Psi}_t^*$$

Solving recursively for $\hat{R}_t^{*,(N-1)} - \hat{R}_t^{(N-1)}$, and substituting it back and leads to

$$E_t(\Delta s_{t+1} + \hat{R}_{t+1}^{*,(N)} - \hat{R}_{t+1}^{(N)} = E_t \sum_{k=0}^{N-1} (\hat{\Psi}_{t+k} - \hat{\Psi}_{t+k}^*)$$

This is very intuitive – the excess return on a carry trade (held to maturity) is the sum of expected future convenience yield differentials. Thus, when operating with 1-period bonds we have equation (D.1), so that only the current convenience yield matters, but when we consider long-term bonds then it is the whole path of expected convenience yield differentials. And since in the model the convenience yield differential changes signs at longer horizons (see Figure 7 for example), the sum $E_t \sum_{k=0}^{N-1} (\hat{\Psi}_{t+k} - \hat{\Psi}_{t+k}^*)$ in fact grows smaller for higher N. Due to their cyclical dynamics that underpin the key results of the model, the convenience yields further into the future cancel out the shorter-horizon ones. In particular, in the benchmark calibration of the model, the sum at horizons of 7 years or more is roughly zero, which matches the data well.

D.4 Term-Structure Effects in UIP violations in the data

We can further examine the empirical evidence on the UIP violations, and decomposes the documented UIP violations into a pure exchange rate effect and a term-structure effect due to violations in the expectations hypothesis (EH) of the interest rate term-structure. The results show that the pure exchange rate component is the primary driver of the estimated UIP violations and their changing nature. This is another reason for why abstracting away from long-term bonds and term structure effects, as I do in the model, is unlikely to be important.

According to the EH, cumulative long-term interest rates are equal to the sum of expected future short-rates over the duration of the long-term interest rate. This implies that a zero coupon n-month bond's cumulative interest rate, $R_t^{(n)}$, is given by

$$R_t^{(n)} = \sum_{k=0}^{n-1} E_t(i_{t+k}),$$

where, as before, i_t is the 1-month interest rate at time t. We can then use this relation to back out risk-neutral expectations of future short-rates from the term-structure itself. Let $i_{t,t+k}$ be the time-t risk-neutral expectation of the 1-month interest rate at time $t+k$, also known as the *forward* interest rate at time t, and note that this is given by the difference in interest rates of a $(k+1)$-months bond and a k-months bond:

$$i_{t,t+k} = R_t^{(k+1)} - R_t^{(k)},$$

Clearly, $i_{t,t} = i_t$, but as has been shown extensively in the bond literature, the EH hypothesis fails at longer horizons (e.g. Campbell and Shiller (1991)), and the forecast errors

$$\eta_{t,t+k} = i_{t,t+k} - i_{t+k}$$

are forecastable by today's (time t) short-rate.

To see how this could affect currency return forecasts, add and subtract the forward interest differential $i^*_{t,t+k} - i_{t,t+k}$ from the excess currency return λ_{t+k+1} to obtain

$$E_t(\lambda_{t+k}) = E_t(\Delta s_{t+k} + i^*_{t,t+k-1} - i_{t,t+k-1}) + E_t(\underbrace{i^*_{t+k-1} - i_{t+k-1} - (i^*_{t,t+k-1} - i_{t,t+k-1})}_{\eta^*_{t,t+k-1} - \eta_{t,t+k-1}}). \tag{D.2}$$

Forecastability in excess currency returns could arise from either of the two components above. The first piece measures how well exchange rates offset forward interest rates, and captures the pure exchange rate effect. In essence, it is the expected excess currency return in a world where the EH holds.[37] The second component measures the forecastability of

[37] This is not a purely theoretical construct, this return can be obtained by going long the excess return

interest-rate excess returns themselves, which captures the term-structure anomaly effect. Next, I decompose the forecastability of excess currency returns into these two components.

To do so, I construct a zero-coupon term-structure of interest rate differentials by using the forward discount at maturities of up to a year, and data on interest rate swaps from Bloomberg for longer maturities. Data on long-maturity interest rates is only available starting in 1990, and the shorter time-series leads me to drop the Euro-legacy currencies from the benchmark results, because they are left with less than 10 years of data. This leaves me with a data on 10 currencies for the period 1990-2013, for which I compute the two components in (D.2) and run separate forecasting regressions on each

$$s_{t+k} - s_{t+k-1} + i^*_{t,t+k-1} - i_{t,t+k-1} = \alpha_{j,k} + \delta_k(i_{j,t} - i^*_{j,t}) + \nu_{j,t+k}$$

$$i^*_{t+k} - i_{t+k} - (i^*_{t,t+k} - i_{t,t+k}) = a_{j,k} + \theta_k(i_{j,t} - i^*_{j,t}) + v_{j,t+k}$$

to estimate δ_k and θ_k, which by construction sum up to the original UIP coefficients β_k

$$\beta_k = \delta_k + \theta_k.$$

Thus, these two series of estimates decompose the UIP violations into a pure exchange rate effect, δ_k, and a term-structure effect, θ_k. The results are plotted in Figure D.1, where the blue line represents the original $\hat{\beta}_k$ estimates (but now estimated on the smaller data set for comparison purposes), the red dash-dot line plots $\hat{\delta}_k$ and the green dashed line plots $\hat{\theta}_k$. The shaded region represents the 95% confidence interval around the estimates of δ_k.

The results show that the exchange rate behavior is the primary driver of the cyclicality in excess currency returns. The $\hat{\delta}_k$ estimates are statistically significant, track $\hat{\beta}_k$ closely and display a very similar pattern across horizons, where they start out negative, and then turn positive at the same time as $\hat{\beta}_k$. In terms of overall magnitudes, the $\hat{\delta}_k$ coefficients account for virtually all of the negative UIP violations at horizons of less than 36 months, and for more than two-thirds of the positive UIP violations at longer horizons.[38] On the other hand, while the term-structure effects are also non-zero and switch from negative to positive, their timing is quite different and the magnitude is much smaller. Thus, the results point to exchange rate behavior as the most important driver of the changing nature of UIP violations, with term-structure effects playing only a secondary role. As such, modeling short-term bonds only is sufficient to understand the first-order features of the puzzle.

D.5 Empirical Debt Dynamics

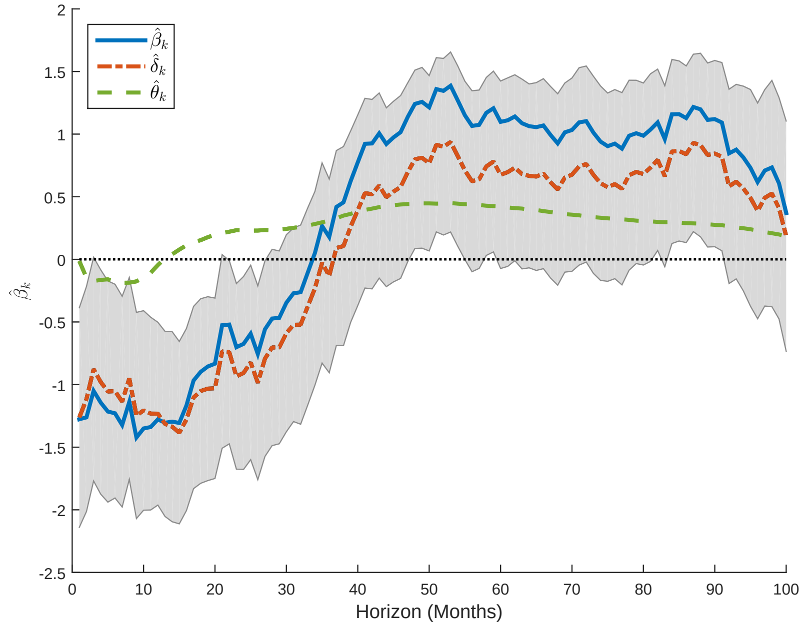
Cyclical debt dynamics are an integral part of the mechanism, and in this section I verify that the data displays non-monotonic dynamics similar to the model. I focus on US government

on a foreign $k+1$ months bond and short the excess return on a k-months foreign bond:

$$\Delta s_{t+k+1} + i^*_{t,t+k} - i_{t,t+k} = s_{t+k+1} - s_t + R_t^{(k+1)*} - R_t^{(k+1)} - (s_{t+k} - s_t + R_t^{(k)*} - R_t^{(k)})$$

[38]While the $\hat{\delta}_k$ estimates barely miss the 95% significance cut-off at 60-80 month horizons, they are significant at the 90% level at all horizons.

Figure D.1: UIP Violations Decomposition



debt, because it is available for the whole sample period at a quarterly basis, while most other foreign government debt series are available only at the annual level before 1991.

I estimate the following AR(p) specification of log government debt:

$$b_t = \mu + \gamma t + \sum_{k=1}^{p} b_{t-k} + \varepsilon_t,$$

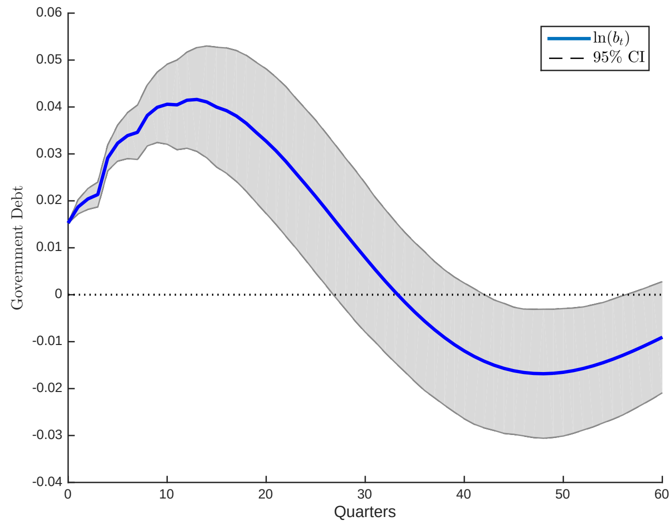
where b_t is the log of US federal debt held by the public (variable FYGFGDQ188S in FRED), and I have included a deterministic time trend, because statistical tests indicate that b_t is trend-stationary. The optimal lag length is 5 according to both the AIC and SIC citeria, hence I set $p = 5$.

I have opted for an univariate AR(p) formulation because I want to remain agnostic about the source of shocks, and rather than try to identify specific structural shocks, I want to estimate the overall dynamics government debt. As we saw in section 5.5.4, the source of shocks does not matter in the model – due to the interaction of monetary and fiscal policy, the dynamics of government debt are determined by complex roots, and thus display cyclicality regardless of the shock. And nevertheless, the results from the AR(p) estimation hold true across a variety of different VAR specifications and identification schemes.

The resulting IRF is plotted in Figure D.2 below. As we can see, in the data US debt dynamics display both momentum and cyclicality, just as in the model. Following a positive shock, debt continues to rise for several quarters before it starts falling back towards steady state. And on the way down, it does not converge monotonically, but dips significantly below its long-run mean before converging.[39]

[39]The results remain the same when using Debt-to-GDP ratio instead of Debt in levels

Figure D.2: Debt Impulse Response



E Debt and Excess Currency Returns Extra Results

Using Debt-To-GDP: Table E.1 below re-estimates the regression specifications of Section 6,

$$\lambda_{j,t+1} = \alpha_j + \beta(i_t - i^*_{j,t}) + \gamma \ln(\text{Debt}_t) + \gamma^* \ln(\text{Debt}^*_t) + \delta \ln(\text{CP}_t) + \text{Additional Controls} + \varepsilon_{j,t+1},$$

using government debt to GDP and Commercial Paper to GDP ratios, as opposed to the variables in levels. All results remain very much the same – the coefficient on US debt variables are negative, large and significant. The coefficients on foreign debt variables are positive, one magnitude smaller and significant in half of the specifications. Thus, the data supports the mechanism of the model, but apportions a significantly bigger role for US debt variables as opposed to foreign liquidity supply.

Quarterly Frequency Results: Table E.2 below re-estimates the regression specifications of Section 6,

$$\lambda^{3m}_{j,t+1} = \alpha_j + \beta(i^{3m}_t - i^{3m,*}_{j,t}) + \gamma \ln(\text{Debt}_t) + \gamma^* \ln(\text{Debt}^*_t) + \delta \ln(\text{CP}_t) + \text{Additional Controls} + \varepsilon_{j,t+1},$$

by using quarterly frequency data only. To match the data frequency, the excess currency returns and the interest rate differentials are for 3-month. The overall results and significance is very similar to the main specifications reported in the main body. The magnitude of the coefficients estimates here is about 3 times as large as the benchmark estimates, as should be expected given that the left-hand side here is 3-month excess returns, whereas it is 1-month excess returns in the daily frequency regressions.

Utilizing longer US data series: Table E.3 below re-estimates the regression specifications of Section 6,

Table E.1: Excess Currency Returns and Debt-to-GDP

	1991 - 2013				1991 - 2007			
	(1)	(2)	(3)	(4)	(1′)	(2′)	(3′)	(4′)
$i_t - i_t^*$	-1.4***	-1.55***	-0.86*	-1.11	-1.83***	-1.95***	-0.83	-0.47
	(0.46)	(0.46)	(0.52)	(0.83)	(0.49)	(0.49)	(0.51)	(0.52)
ln(Debt)		-0.48	-3.28***	-5.62***		-1.64***	-5.69***	-5.00***
		(0.38)	(1.22)	(1.52)		(0.59)	(1.41)	(1.89)
ln(Debt*)		0.18	0.27**	0.18		0.08	0.12	0.22**
		(0.11)	(0.12)	(0.13)		(0.17)	(0.11)	(0.11)
ln(CP)			-2.76**	-5.04***			-3.52***	-2.28
			(1.12)	(1.51)			(1.08)	(1.84)
Add. Controls	No	No	No	Yes	No	No	No	Yes
# Currencies	10	10	10	10	10	10	10	10
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Estimates with Driscoll and Kraay (1998) standard errors robust to heteroskedasticity, serial correlation and cross-equation correlation. The debt stock variables are exponentially detrended. ***, ** and * denote significance at the 1%, 5% and 10% level respectively. The excess currency returns (LHS variable) are expressed in terms of percent.

Table E.2: Excess Currency Returns and Debt, Quarterly Frequency

	1991 - 2013				1991 - 2007			
	(1)	(2)	(3)	(4)	(1′)	(2′)	(3′)	(4′)
$i_t^{3m} - i_t^{3m,*}$	-1.2***	-1.16***	0.13	0.27	-1.36**	-1.51	0.13	1.93
	(0.48)	(0.97)	(1.53)	(2.13)	(0.53)	(1.09)	(1.84)	(2.22)
ln(Debt)		-3.41	-17.60**	-15.42**		-5.28*	-24.89***	-29.86***
		(2.96)	(7.68)	(7.95)		(3.19)	(9.12)	(10.01)
ln(Debt*)		0.35	0.28	-0.17		0.35	0.22	0.23
		(0.78)	(0.81)	(0.66)		(0.88)	(0.91)	(0.75)
ln(CP)			-6.95**	-6.72*			-10.58**	-16.11**
			(3.45)	(3.67)			(4.44)	(6.38)
Add. Controls	No	No	No	Yes	No	No	No	Yes
# Currencies	10	10	10	10	10	10	10	10
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Estimates with Driscoll and Kraay (1998) standard errors robust to heteroskedasticity, serial correlation and cross-equation correlation. The debt stock variables are exponentially detrended. ***, ** and * denote significance at the 1%, 5% and 10% level respectively. The excess currency returns (LHS variable) are expressed in terms of percent.

$$\lambda^{3m}_{j,t+1} = \alpha_j + \beta(i^{3m}_t - i^{3m,*}_{j,t}) + \gamma \ln(\text{Debt}_t) + \delta \ln(\text{CP}_t) + \text{Additional Controls} + \varepsilon_{j,t+1},$$

by making use of the longer availability of US data for government debt and commercial paper. Thus, the data for those regressions starts in 1984, the earliest availability of USD commercial paper data. By necessity, the regressions exclude foreign debt due to the lack of data going back to 1984, however the additional controls vector, still includes foreign stock market volatility and yield slope. Lastly, I can now also safely include all 18 currencies, as we have at least 15 years of data for each currency pair.

All results remain the same as before, both quantitatively and qualitatively. We still see large and significant negative coefficient values on US debt, and similarly larger effects in the pre-crisis period.

Table E.3: Excess Currency Returns and Debt, 1984 (US debt only)

	1984 - 2013				1984 - 2007			
	(1)	(2)	(3)	(4)	(1′)	(2′)	(3′)	(4′)
$i_t - i^*_t$	-0.98***	-1.06***	-0.95**	-1.74***	-1.07***	-1.24***	0.91**	-1.16**
	(0.40)	(0.40)	(0.41)	(0.53)	(0.41)	(0.41)	(0.44)	(0.46)
ln(Debt)		-1.16*	-2.21**	-1.96**		-2.09***	-3.31***	-3.07***
		(0.7)	(0.94)	(0.91)		(0.70)	(0.91)	(0.94)
ln(CP)			-0.67*	-0.48			-1.28**	-1.11
			(0.37)	(0.37)			(0.63)	(0.79)
Add. Controls	No	No	No	Yes	No	No	No	Yes
# Currencies	18	18	18	18	18	18	18	18
Fixed Effects	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes

Estimates with Driscoll and Kraay (1998) standard errors robust to heteroskedasticity, serial correlation and cross-equation correlation. The debt stock variables are exponentially detrended. ***, ** and * denote significance at the 1%, 5% and 10% level respectively. The excess currency returns (LHS variable) are expressed in terms of percent.